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Three Cases of Nonlinear Evolution Theory of Ultrasound Propagation in Liquids Containing Many Microbubbles with a Polydispersity of Bubble Size

大きさの異なる気泡を含む液体中の超音波伝播：
3 通りの媒質に対応する非線形発展理論

Takuma Kawame[1‡], Tetsuya Kanagawa[2], and Reona Ishitsuka[3] ([1]Coll. Eng. Syst., Univ. of Tsukuba; [2]Fac. Eng., Inf. & Syst., Univ. of Tsukuba; [3]Grad. Sch. Syst. Inf. Eng., Univ. of Tsukuba)
川目拓磨 [1‡], 金川哲也 [2], 石塚怜央奈 [3] ([1]筑波大工シス, [2]筑波大シス情, [3]筑波大院シス情)

1. Introduction

Recently, an ultrasound diagnosis has been established and utilized. The use of nonlinear oscillations of contrast agent microbubbles (e.g., Sonazoid, Levovist, etc.) significantly improves the resolution of image (e.g., [1]). Such microbubbles should be produced as quite small compared with a diameter of blood vessel. However, an error of diameter of such a small microbubble cannot be avoided in the production process.

Although nonlinear propagation of ultrasound in liquids containing many microbubbles has long been theoretically studied, initial radius of microbubbles has been treated as the same (e.g., [2]). From the viewpoint of contrast agent microbubble production, a small error of diameter of microbubbles should be incorporated in theories.

The purpose of this study is to investigate nonlinear propagation of ultrasound in bubbly liquids with a small initial polydispersity of bubble size. Especially, we do not assume explicitly function form of initial bubble radius toward an utilization into broad applications.

2. Theoretical Analysis

2.1 Problem and assumption

One-dimensional propagation of pressure wave (or ultrasound) in initially quiescent liquids uniformly containing many spherical microbubbles with an initial polydispersity of bubble size is theoretically studied. The polydispersity appears in a field far from the sound source. Bubble oscillations are spherically symmetric, and bubbles do not coalesce, break up, appear, and disappear.

2.2 Basic equations

The set of basic equations based on a two-fluid model [3], which is composed of the conservation laws of mass and momentum for gas and liquid phases, the equation of bubble oscillations (Keller equation [4]), the equation of state for both phases, and so on, is used. The other equations are not shown here for the economy of space (see the explicit form in our previous papers [2, 5]).

2.3 Singular perturbation analysis

All the dependent variables are expanded as power series of small but finite amplitude, ϵ ($0 < \epsilon \ll 1$), as a nondimensional perturbation [6]. We consider an initial small polydispersity (i.e., nonuniformity). The bubble radius as dependent variable is then expanded as follows:

$$R^*/R_0^* = \begin{cases} 1+\epsilon(R_1+\delta_R) \\ \qquad +\epsilon^2 R_2 + O(\epsilon^3)\ \text{(KdVB)} \\ 1+\epsilon R_1+\epsilon^2(R_2+\delta_R) \\ \qquad +\epsilon^3 R_3 + O(\epsilon^4)\ \text{(NLS)} \end{cases} \tag{3}$$

where R^* is the radius of a representative bubble, δ_R is a known function representing the initial polydispersity; the subscript 0 denotes initial unperturbed state and the superscript * does dimensional quantity. Noting that the expansion of void fraction is immediately determined from Eq. (3).

The expansions of the other dependent variables are the same as those in our previous study [2].

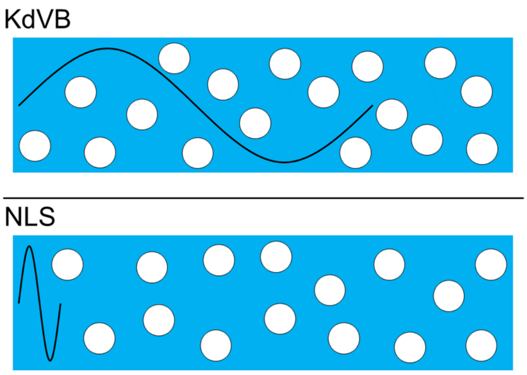

Fig. 1 Difference between length scale in KdVB case and that in NLS case.

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[1] s1711125@s.tsukuba.ac.jp
[2] kanagawa.tetsuya.fu@u.tsukuba.ac.jp

2.4 Parameter scaling

The size of set of three nondimensional ratios is determined as in our previous study [2]:

$$\left(\frac{U^*}{c_{\mathrm{L0}}^*}, \frac{R_0^*}{L^*}, \frac{\omega^*}{\omega_{\mathrm{B}}^*}\right) = \begin{cases} \left(O(\sqrt{\epsilon}), O(\sqrt{\epsilon}), O(\sqrt{\epsilon})\right) & \text{(KdVB)} \\ \left(O(\epsilon^2), O(1), O(1)\right) & \text{(NLS)} \end{cases} \tag{4}$$

where U^* is a typical propagation speed of waves, c_{L0}^* is the sound speed in pure water, L^* is a typical wavelength, ω^* is an incident frequency of waves, and ω_{B}^* is the eigenfrequency of single bubble oscillations.

As shown in Eq. (4), in the case of KdVB equation, the typical propagation speed of waves is small compared with the sound speed in pure water, the typical wavelength is large compared with the initial bubble radius, and the incident frequency of waves is low compared with the eigenfrequency of waves. On the other hand, in the case of the NLS equation, the typical propagation speed of waves is considerably small compared with the sound speed in pure water, the typical wavelength is comparable with the initial bubble radius, and the incident frequency of waves is comparable with the eigenfrequency of waves.

3. Result

3.1 Case A: long wave and large polydispersity [7]

For low frequency long wave, we have the following resultant equation from the second-order of approximation:

$$\frac{\partial R_1}{\partial \tau} + \Pi_1 R_1 \frac{\partial R_1}{\partial \xi} + \Pi_2 \frac{\partial^2 R_1}{\partial \xi^2} + \Pi_3 \frac{\partial^3 R_1}{\partial \xi^3} = 0, \tag{5}$$

via variables transform,

$$\tau = \epsilon t, \tag{6}$$

$$\xi = x - (1 + \epsilon \Pi_0 + \Pi_4(\delta_R))t, \tag{7}$$

where $\Pi_i (i = 0,1,2,3)$ is the constant coefficient and the same as those in the previous study [2] and Π_4 is the variable coefficient depending on $\delta_R(\epsilon x)$. As is clear from Eq. (7), the small polydispersity then affects the advection of waves at the far field.

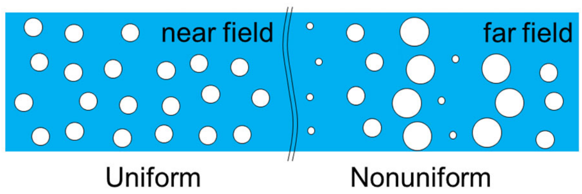

Fig. 2 Conceptual diagram of the result in Sec. 3.1: near field is initially monodisperse and far field is initially polydisperse.

3.2 Case B: short wave and small polydispersity

For high frequency short wave, we have the following equation from the third-order of approximation:

$$\mathrm{i}\frac{\partial A}{\partial \tau} + \frac{1}{2}\frac{\mathrm{d}v_{\mathrm{g}}}{\mathrm{d}k}\frac{\partial^2 A}{\partial \xi^2} + \nu_1 |A|^2 A + \nu_2 A = 0, \tag{8}$$

via variables transform,

$$\tau = \epsilon^2 t, \tag{9}$$

$$\xi = \epsilon(x - v_{\mathrm{g}} t) + \epsilon^2 \frac{\nu_3(\delta_R)}{K} t, \tag{10}$$

where A is the complex amplitude of the envelope wave, v_{g} group velocity, k wavenumber of the envelope wave. Although ν_1 and ν_2 are the real constant coefficients [2], $\nu_3(\delta_R)$ is the variable coefficient depending on $\delta_R(\epsilon^2 x)$. As in Sec. 3.1, polydispersity contributes wave advection at the far field.

3.3 Case C: short wave and large polydispersity

We here briefly introduce a further perspective. For the NLS case in Eq. (3), by beginning the expansion from $\epsilon(R_1 + \delta_R(\epsilon^2 x))$, we can treat larger polydispersity than Sec. 3.2. The detail will be shown in presentation.

4. Summary

Pressure wave propagation in initially quiescent water uniformly containing many spherical microbubbles with an initial small polydispersity was theoretically studied and the resultant nonlinear wave equations were derived. The polydispersity contributes the wave advection at the far field.

Acknowledgment

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