Development of a wave machine to model phononic band gaps

ウェーブマシンによるフォノニックバンドギャップの再現

Tetsu Omiya[‡], Motonobu Tomoda, Masahiro Ino, Kentaro Fujita, Osamu Matsuda, and Oliver B. Wright (Grad. School Eng., Hokkaido Univ.)

大宮 哲[‡], 友田 基信, 猪野 真大, 藤田 健太郎, 松田 理, Oliver B. Wright (北大院 工)

1. Introduction

Techniques to block waves in specific frequency ranges are in wide demand. For acoustic waves in periodic media, such frequency ranges are called phononic band gaps. Two main methods to produce such band gaps by static structures exist. One way is to use phononic crystals, i.e. acoustic periodic structures, and the other is to use acoustic metamaterials, i.e. structures with local vibrational resonances. To bring out the basic physics of such phenomena for instructive purposes, experiments on mechanical structures of human dimensions have been recently developed [1,2]. But in these systems the detailed mechanical motion is not immediately visually accessible. We are therefore developing mechanical wave-supporting structures based on the Shive wave machine, which we previously used for modeling phononic crystals [3]. Here, we present an upgraded wave machine, and demonstrate the creation of phononic band gaps not only in phononic crystals but also in acoustic metamaterials.

2. Shive wave machine

The Shive wave machine is suitable for demonstrating various wave phenomena because of the large vibrational amplitude, small wave velocity and the ability to observe wave propagation visually [4]. The structure consists of a periodic onedimensional array of torsionally-coupled rods suspended on wires. When a rod rotates, the tension of the wire generates a restoring force on the adjacent rods, so the Shive wave machine can exhibit wave phenomena such as reflection, transmission, and standing waves.

The properties of the Shive wave machine as a wave-supporting medium are determined by the moment of inertia of each rod I and by the torsional



Fig. 1 Correspondence between the wave machine and a one-dimensional mass-spring model. On rod rotation, the tension in the wires T generate restoring forces F.

constant $\kappa = 2Td^2/a$, where T is wire tension, d is the distance from the center of the rods to the two wires and a is the lattice constant (see Fig. 1). The equation of motion for the *n*-th rod is

$$H\ddot{\theta}_n = \kappa(\theta_{n+1} + \theta_{n-1} - 2\theta_n), \qquad (1)$$

where θ_n is the torsion angle of the *n*-th rod. The system represents an analogy of a one-dimensional mass-spring model. The equation of the motion of the equivalent mass-spring system is

$$m\ddot{u}_n = K(u_{n+1} + u_{n-1} - 2u_n), \qquad (2)$$

where u_n is the displacement of the *n*-th mass point, *m* is the mass, and *K* is the spring constant. Comparing Eqs. (1) and (2), the moment of inertia *I* and torsional constant κ in the Shive wave machine correspond to the mass *m* and spring constant *K* in an equivalent one-dimensional mass-spring model, respectively.

3. Phononic band gap from spatial periodicity

The phononic band gap arises from the spatial periodicity, as shown in **Fig. 2** for the discrete mechanical mass-spring model. The calculation of the dispersion relation from this model is commonly



Fig. 2 Schematic diagram of a periodic mass-spring model and its dispersion relation. Red solid lines are the phononic branches for real wavevector k, and purple dotted lines are in the phononic band gap for imaginary k, implying spatial wave decay.

given in chapters on phonons in solid-state physics textbooks. Using a unit cell in the Shive wave machine with two different moments of inertia, it is therefore possible to mimic the behavior of phononic crystals [3].

4. Phononic band gap from local oscillators

The phenomenon of phononic band gaps arising from arrays of local oscillators, i.e. acoustic metamaterials, is shown for the discrete mechanical mass-spring model in **Fig. 3**. Each outer mass has an inner oscillator. Band gaps are created near the resonant frequency of the inner oscillator [5].

We mimic acoustic metamaterials with the Shive wave machine by attaching plastic blade springs, acting as local oscillators, with a resonant frequency of 3.5 Hz to both ends of each rod. Fig. 4 shows frames from a movie of the torsional wave propagation. Waves can propagate in the metamaterial part of the wave machine when the incident wave has a somewhat lower or higher frequency than the resonant frequency of the plastic blade, as shown in Fig. 4 (a) and (c), but cannot propagate at a frequency in the band gap, as shown as Fig. 4(b). In the band gap, waves only penetrate to a distance of one or two rods.

4. Conclusion

We have demonstrated phononic band gaps arising from spatial periodicity and also from local oscillators using a Shive wave machine. This e represents an excellent mechanical platform not only for educational purposes but also for demonstrating new mechanisms to control wave motion, including active control.

References

- 1. G. Ma and P. Sheng: Science Advances 2 (2016) e1501595.
- 2. K, Bertoldi et al.: Nat. Rev. 2 (2017) 17066.
- 3. M. Ino, M. Tomoda, and O. B. Wright: Proc. USE2018, 1J4-2.
- J. N. Shive: Similarities in Wave Behavior, Williams & Wilkins Co. Series, Baltimore, Md., 1961.
- 5. S. Lee and O. B. Wright: Phys. Rev. B 93 (2016) 024302.



Fig. 3 Schematic diagram of an elastic metamaterial based on a mass-spring model and its dispersion relation. Red solid lines are the phononic branch, and purple dotted lines are in the phononic band gap. The dotted blue curves indicate the phononic branches without inner masses, and the horizontal dotted blue line the resonant angular frequency of the inner mass ($\omega_0 = \sqrt{K_2/m_2}$).



Fig. 4 Frames for constant-frequency movies in which the wave propagates from a non-metamaterial zone (left) to a metamaterial zone (right) (a) below the band gap, at 1.5 Hz, (b) in the band gap at 3.5 Hz (i.e. near the resonance of the blade springs), and (c) above the band gap, at 5 Hz. The waves are generated with a motor and crank on the left-hand side. To form the acoustic metamaterial, blade springs are added at both ends of each rod.