

Moving sound source with arbitrary trajectory in two-dimensional finite difference-time domain method

2次元 FDTD 法における任意軌道を伴う移動音源

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1. Introduction

Finite difference-time domain (FDTD) method [1] is a most popular numerical method for sound field analysis. In most cases in the FDTD calculation, sound source and receiver are fixed, then the response between them is mainly calculated. However, to analyze the noise of automobiles and trains, or bat sonar mechanism, it is necessary to implement moving sound sources in FDTD method.

In this paper, two methods are proposed as a method of implementing a moving sound source with arbitrary trajectory in the two-dimensional FDTD method. One is the direct method in which source waveform is radiated while switching grid points on the moving path to be driven at every time step. The other is a convolution method in which all impulse responses from the sound source position at every time step are calculated, then the source waveform is convoluted while switching the impulse response according to the sound source movement. Formulation and numerical experiments are carried out for a two-dimensional sound field. The numerical accuracy will be compared between the direct method and the convolution method.

2. Theory

2.1 Direct method

In order to implement a moving sound source in the FDTD method, the grid points on the moving path of the sound source are driven switching every time step according to the sound source position. In the two-dimensional case, when a sound source is located between grid points, four adjacent grid points are driven according to the source weighting functions. When the sound source is located at $(x, y) = (x_i + d_x, x_j + d_y)$ as shown in Fig. 1 (a), the position of the sound source on the local coordinate system is expressed as $(\xi_i, \eta_j) = (2d_x/\Delta - 1, 2d_y/\Delta - 1)$ as shown in Fig. 1 (b), where Δ is grid interval. The source weighting functions are given as follows

$$\begin{aligned} w_1 &= (1 - \xi_i)(1 - \eta_j)/4, w_2 = (1 + \xi_i)(1 - \eta_j)/4 \\ w_3 &= (1 + \xi_i)(1 + \eta_j)/4, w_4 = (1 - \xi_i)(1 + \eta_j)/4 \end{aligned} \quad (1)$$

2.2 Convolution method

We here consider a discrete-time system.

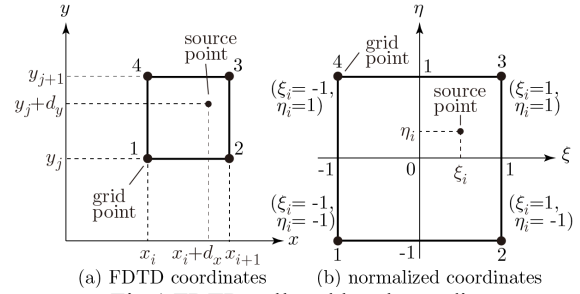


Fig.1 FDTD cell and local coordinates.

When sound source and receiving point are fixed, the acoustic signal p received at the receiving point is expressed as

$$p(k) = \sum_{m=0}^k s(m)h(k - m) \quad (2)$$

where $s(m)$ is sound source signal and $h(k)$ is impulse response. In the case of the moving sound source, the positional relationship between the sound source and the receiving point changes every time step, so that the impulse response $h(k)$ also changes accordingly. When the sound source is located at the position $\mathbf{r}(k)$ at a certain discrete time k , the impulse response radiated from that sound source position is expressed as $h(k, \mathbf{r}(k))$. Therefore the receiving signal is expressed as

$$p(k) = \sum_{m=0}^k s(m)h(k - m, \mathbf{r}(m)) \quad (3)$$

Thus, in order to obtain the signal $p(k)$ at the receiving point, it is necessary to obtain $h(k, \mathbf{r}(m))$ at all sound source positions $\mathbf{r}(m)$ on the moving path in advance which can be easily obtained by the use of reciprocity.

3. Numerical experiments

Numerical experiments are performed by the CE-FDTD (IWB) method [2,3]. Figure 2 shows the numerical model of linear trajectory case. The grid size is $\Delta=10$ mm, time step is $\Delta t=29.4$ μ s, and sound speed is $c_0 = 340$ m/s, so the Courant number χ is 1. The region is divided into $5,000 \times 5,000$ FDTD cells. The boundary condition is Higdon's second order absorbing boundary. The sound source is a point source passing linearly in front of the receiving point with initial speed of $M =$

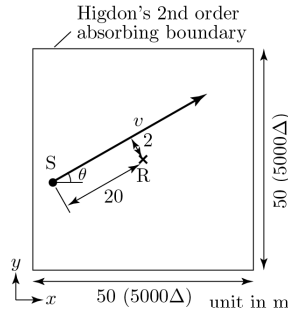


Fig.2 Numerical model for moving sound source.

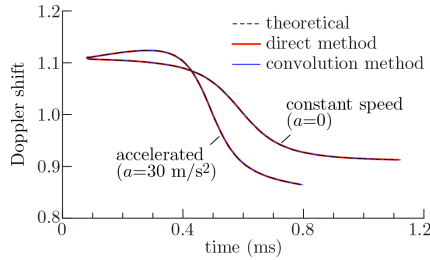


Fig.3 Doppler shift for passing sound source.

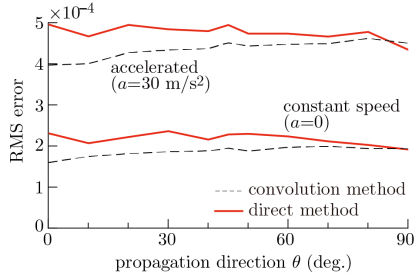


Fig.4 RMS error against propagation direction.

0.1 and acceleration of 0 or 30 m/s². The source radiates a continuous sine wave with a frequency of 500 Hz. In the convolution method, impulse responses are calculated in advance between the grid points on the moving path and the sound receiving point.

Figure 3 shows the result of short-time Fourier transform of the sound pressure waveform calculated at the receiving point. In the figure, the center frequency of the power spectrum is calculated by the reassignment method. The calculated result and the theoretical result show good agreement for each method. Figure 4 shows the root mean square error against the propagation direction θ . The error is larger for the accelerated case and is within 5×10^{-4} in all directions. It is found that it is possible to calculate a moving source even when the Doppler frequency changes continuously.

Figure 5 shows the numerical model of quadratic trajectory case. The calculation conditions other than the sound source trajectory are the same as the linear trajectory case. Figure 6 shows the result of short-time Fourier transform of the sound pressure waveform calculated at the receiving point. In the quadratic case, the calculated result and the theoretical result again

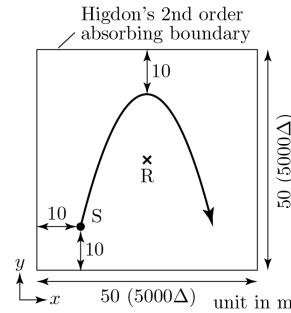


Fig.5 Numerical model for quadratic trajectory.

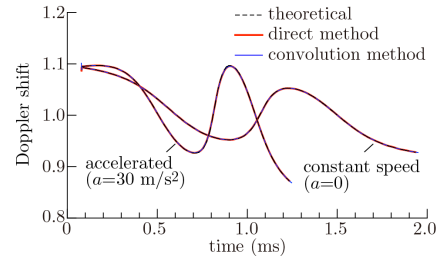


Fig.6 Doppler shift for quadratic trajectory case.

show good agreement for each method. The RMS error is tabulated in Table 1. As with the linear trajectory, the error of the accelerated case is larger than the constant speed case. It is found that it is possible to calculate a moving source with the arbitrary trajectory.

Table 1 RMS error for quadratic trajectory case.

method	direct	convolution
constant speed	3.4×10^{-4}	2.2×10^{-4}
accelerated	8.3×10^{-4}	6.9×10^{-4}

Figure 7 shows the calculation time against the sound source speed for the linear trajectory case. The convolution method can be calculated in the same time without depending on the source speed, but the direct method shows that the calculation time increases in inverse proportion to the moving speed. It is found that the convolution method can be calculated faster at most source speeds.

References

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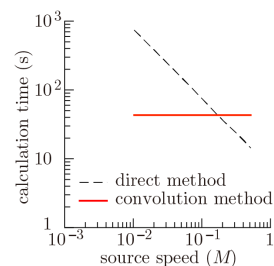


Fig.7 Calculation time against source speed.