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Simulation of shear wave propagation near heterogeneous tissue surface by using velocity-stress FDTD method

速度-応力時間領域差分法を用いた不均質な組織表面近傍の剪断波伝搬シミュレーション

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1. Introduction

Recently, new techniques about elasticity measurement using high-frequency airborne ultrasound[1)] and optical coherence tomography (OCT) system[2)] have been studied. The techniques use surface waves. In general, heterogeneous medium such as human tissues affect measurement accuracy, however, few studies about shear wave propagation phenomena in heterogeneous mediums have been reported. In Refs.[2, 3)], measurement results show that surface wave speeds become smaller than actual due to a copper wire inserted in an agar sample. Since, experimentations tend to be complicated (production of samples and experimental systems), we applied velocity-stress finite-difference method[4)] in order to investigate the shear wave propagation near heterogeneous tissue surface.

2. Simulation

2.1 Velocity-stress FDTD method

Navier-stokes equations expressed in Eqs. (1)-(3) were used in 2-D finite-difference time-domain (FDTD) simulation[6, 7)]. In Eqs. (1)-(3), v is particle velocity, ρ is mass density, μ is elastic shear constant, σ is stress tensor, and Δx and Δy are grid intervals.

$$v_x{}^{n+1}|_{i,j} = v_x{}^{n}|_{i,j} + \frac{\Delta t}{\rho \Delta x}\left(\sigma_{xy}^{n}|_{i,j+\frac{1}{2}} - \sigma_{xy}^{n}|_{i,j-\frac{1}{2}}\right) \quad (1)$$

$$v_y{}^{n+1}|_{i,j} = v_y{}^{n}|_{i,j} + \frac{\Delta t}{\rho \Delta y}\left(\sigma_{xy}^{n}|_{i,j+\frac{1}{2}} - \sigma_{xy}^{n}|_{i,j-\frac{1}{2}}\right) \quad (2)$$

$$\begin{aligned}\sigma_{xy}^{n}|_{i,j+\frac{1}{2}} = \sigma_{xy}^{n}|_{i,j+\frac{1}{2}} &+ \frac{\mu\Delta t}{\Delta y}\left(v_x{}^{n+1}|_{i,j+1} - v_x{}^{n+1}|_{i,j+1}\right) \\ &+ \frac{\mu\Delta t}{\Delta x}\left(v_y{}^{n+1}|_{i+\frac{1}{2},j+\frac{1}{2}} - v_y{}^{n+1}|_{i-\frac{1}{2},j+\frac{1}{2}}\right) \\ &+ \frac{\eta}{\Delta y}\left(v_x{}^{n+1}|_{i,j+1} - v_x{}^{n+1}|_{i,j+1}\right) \\ &- \frac{\eta}{\Delta y}\left(v_x{}^{n}|_{i,j+1} - v_x{}^{n}|_{i,j}\right) \\ &+ \frac{\eta}{\Delta x}\left(v_y{}^{n+1}|_{i+\frac{1}{2},j+\frac{1}{2}} - v_y{}^{n+1}|_{i-\frac{1}{2},j+\frac{1}{2}}\right) \\ &- \frac{\eta}{\Delta x}\left(v_y{}^{n}|_{i+\frac{1}{2},j+\frac{1}{2}} - v_y{}^{n}|_{i-\frac{1}{2},j+\frac{1}{2}}\right) \end{aligned} \quad (3)$$

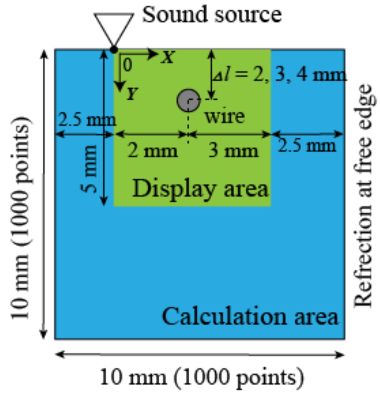

Fig. 1 Simulation setup of shear wave propagation near copper wire. Δl (=2, 3, 4 mm) is the depth from surface.

2.2 Simulation of shear wave propagation near heterogeneous tissue surface

We investigated the surface wave propagation characteristics of an agar gel with a copper wire by velocity-stress FDTD simulation of shear wave near heterogeneous tissue surface. Fig. 1 shows the coordinate axes and the simulation setup. The copper wire with a diameter of 0.2 mm was used as a structure and the depth Δl from the surface was changed by 2, 3, and 4 mm. The parameters of simulation were set as shear wave speed $c = 5$ m/s, spatial grid intervals $\Delta h = 1.0 \times 10^{-5}$ m, time interval $\Delta t = 1.0 \times 10^{-6}$ s, and courant constant $\alpha = 0.5$. Young's moduli E of 75 kPa and viscosity η of 0.022 Pa·s were used, where $E = 3\mu$ and $E = 3\rho c^2$.

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Stress was applied with burst sine wave (2 kHz, 3 waves) to heterogeneous medium. Particle velocity and stress were alternately calculated with the calculation time of 2 ms and the measurement interval of 0.5 mm. Then, the surface wave speeds were calculated.

3. Result and discussion

Figs. 2(a) and (b) show the propagation of shear wave at t = 0.7 and 1.3 ms, obtained by visualizing the particle velocity of the simulation results with a mesh plot. Fig. 2 also shows that the shear wave is reflected when the shear waves pass through the copper wire.

Fig. 3 shows the calculation results of surface wave speeds. When the depth was Δl = 2 mm, the speed of surface wave at x = 1.25 mm became smaller than those at other positions. The results show that the surface wave speeds are affected by internal structure. The trends of calculation results were approximately consistent with the trends of the experimental values[3, 4] plotted in Fig. 3.

Generally, surface wave velocity is known to be affected by structures, when the structures are positioned less than one wavelength below the surface. Since the excitation frequency was 2 kHz and c = 5 m/s in this calculation, the wavelength of the shear wave was 2.5 mm. Therefore, the effect on the surface wave was the biggest at Δl = 2 mm, which was within the one wavelength. On the other hand, as the position of the copper wire deepened, the effect on the surface waves decreased. At the deepest point of Δl = 4 mm, the effect was almost non-existent.

4. Conclusion

Shear wave propagation near heterogeneous tissue surface was successfully visualized by velocity-stress FDTD simulations. From the calculation results, effectiveness of the FDTD simulation to evaluate measurement methods of elastic properties near heterogeneous tissue is indicated.

Reference

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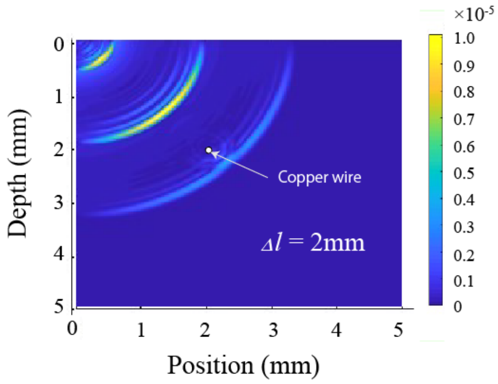
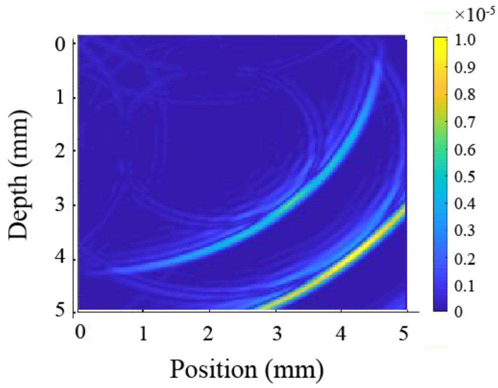


(a) t = 0.7 ms



(b) t = 1.3 ms

Fig. 2 The result of the FDTD simulations with a mesh plot of the particle velocity when $\Delta l = 2$ mm. Initial stress value $\sigma_{xy} = 1$ was applied.

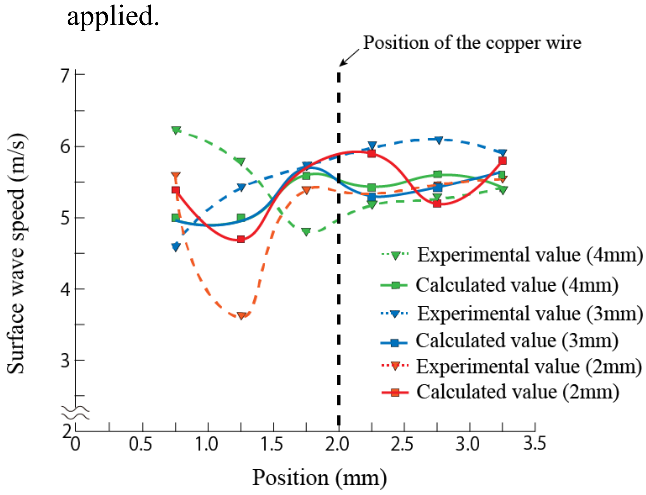


Fig. 3 Comparison of experimental value and calculated value of surface wave speed at depths of 2, 3 and 4 mm.