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Theoretical Study on Nonlinear and Thermal Effects of High-Speed Pressure Waves in Bubbly Liquids
気泡含有水中を水中音速超で伝わる高速圧力波の
非線形性と熱に着目した理論解析

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1. Introduction

Dispersion effect of waves is one of the most important properties of pressure (or shock) wave propagation in bubbly liquids and does not appear for the case of single phase fluids. Although the wave frequency is independent of the wavelength in single phase fluids, the wave frequency is dependent on the wavelength in bubbly liquids. Figure 1 shows the conceptual diagram of linear dispersion relation in bubbly liquids shows that there exist two branches, i.e., Slow mode and Fast mode [1, 2]. Slow mode and Fast mode correspond to the phase velocity always lower and higher than the speed of sound in pure liquid, respectively.

Fast mode appears only for the case that the compressibility of liquid phase is taken into account. Although Slow mode was found about 50 years ago and has a long history of theoretical studies, few studies have been carried out for Fast mode (e.g., [1, 2]). Waves propagating at a speed close to the speed of sound in pure water ahead of shock waves were observed, but since the observed waves have small amplitude [3], it is implied that a theoretical prediction is significantly desired. Since its amplitude is small but finite, a weakly nonlinear analysis (i.e., neither linear nor strongly nonlinear analyses) [4] is appropriate for the prediction.

The aim of this paper is a theoretical prediction of shock wave in Fast mode. Especially, we incorporate the thermal effect [5, 6], which was neglected in our preceding study [7]. As a result, we found that strong dissipation effect appears and an interaction between nonlinearity and thermophysical property.

2. Problem statement

Weakly nonlinear propagation of plane progressive pressure waves in an initially quiescent compressible water uniformly containing many spherical microbubbles is theoretically investigated and formulated as nonlinear wave equation. We summarize the main assumptions: (i) Incident wave frequency is quite higher than eigenfrequency of bubble oscillations; (ii) Wavelength is quite shorter than the bubble radius; (iii) Gas inside bubbles is only composed of non-condensable gas, and hence the phase change across gas-liquid interface does not occur; (iv) Viscosity for bubbly liquids is considered by using the initial void fraction and the viscous coefficient of liquid phase.

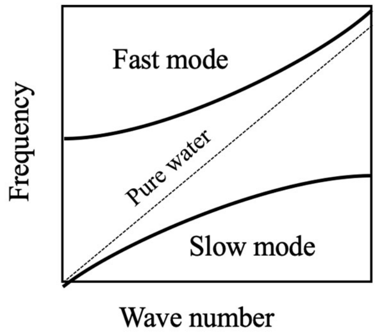

Fig. 1 Linear dispersion relation of pressure waves in a compressible liquid containing many bubbles [1, 2].

3. Basic equations

We newly introduce an energy equation incorporating a thermal conduction at the bubble-liquid interface [5, 6]:

$$\frac{\mathrm{D}}{\mathrm{D}t^*}\left(p_{\mathrm{G}}^* R^{*3\gamma}\right) = 3(\gamma-1)\lambda_{\mathrm{G}}^* R^{*3\gamma-1}\frac{T_{\mathrm{G0}}^* - T_{\mathrm{G}}^*}{\sqrt{2\pi D_{\mathrm{G}}^*/\omega_{\mathrm{B}}^*}}, \tag{1}$$

where t^* is the time, p^* pressure, R^* bubble radius, γ ratio of specific heats in the gas phase, λ_{G}^* thermal conductivity in gas, T^* temperature, D_{G}^* thermal diffusivity in gas, ω_{B}^* eigenfrequency of single bubble oscillations; the subscript G denotes volume-averaged variables in gas phases, subscript 0 does the quantities in the initial uniform state at rest, and superscript * does a dimensional quantity.

The Keller equation for bubble dynamics as spherical symmetric oscillations in a compressible liquid is used (shown only important terms):

$$\rho_{\mathrm{L0}}^*\left[R^*\frac{\mathrm{D}R^*}{\mathrm{D}t^*} + \frac{3}{2}\left(\frac{\mathrm{D}R^*}{\mathrm{D}t^*}\right)^2\right] = P^* + \frac{R^*}{c_{\mathrm{L0}}^*}\frac{\mathrm{D}}{\mathrm{D}t^*}\left(p_{\mathrm{L}}^* + P^*\right), \tag{2}$$

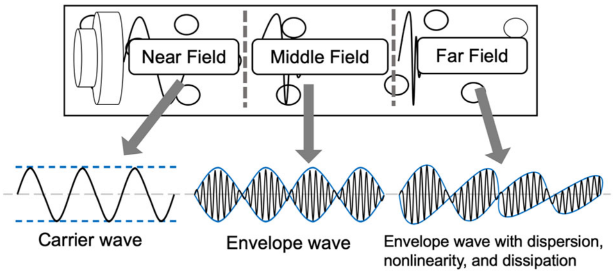

Fig. 2 Conceptual illustration of division of sound field into three fields and asymptotic behaviors of carrier and envelope waves.

where ρ^* is the density, P^* surface-averaged liquid pressure at gas-liquid interface, c_{L0}^* sound speed in pure water; the subscript L denotes volume-averaged variables in liquid phases. Further, the conservation laws of mass, momentum, and number density of bubbles, Tait equation of state for liquid, equation of state for ideal gas inside bubble, conservation equation of mass inside bubble, and balance of normal stresses at the bubble-liquid interface are also used (see, the explicit forms in [4]).

4. Results

All the dependent variables are expanded in a power series, e.g., the expansion of R^* is

$$R^*/R_0^* = 1 + \epsilon R_1 + \epsilon^2 R_2 + \cdots, \tag{3}$$

where the perturbation $\epsilon\,(\ll 1)$ is a typical nondimensional (finite but small) amplitude. Further, we assume the solution of R_1 into the following form:

$$R_1 = A\exp[i(kx - \Omega t)] + \mathrm{c.c.}, \tag{4}$$

where the envelope A is the complex amplitude depending on slow scales, k is the wavenumber, Ω is nondimensional frequency, t is the nondimensional time and x is the nondimensional space coordinate.

4.1 Near Field

Leading order of approximation gives

$$A = \mathrm{const.} \tag{5}$$

That is, the envelope wave is regarded as a constant.

4.2 Central Field

Second order of approximation gives

$$\frac{\partial A}{\partial t} + v_{\mathrm{g}}\frac{\partial A}{\partial x} + \epsilon(D_1 A + iD_2 A) = 0, \tag{6}$$

where v_{g} is the group velocity and D_i is the real constant. That is, the envelope wave is almost constant along characteristics with respect to v_{g} with a small effect of third- and fourth-terms in Eq. (6).

4.3 Far Field

Deriving third order of approximation and combining the near, center, and far fields give the following NLS (Non-Linear Schrödinger) type evolution equation [7] (Fig. 2):

$$\begin{aligned} i\frac{\partial A}{\partial \tau} + \frac{1}{2}\frac{\mathrm{d}v_{\mathrm{g}}}{\mathrm{d}k}\frac{\partial^2 A}{\partial \xi^2} + (\nu_{11} + i\nu_{12})|A|^2 A \\ + (\nu_{21} + i\nu_{22})iA + \nu_{31}\frac{\partial A}{\partial \xi} \\ = 0 \end{aligned} \tag{7}$$

with variables transform,

$$\begin{aligned} \tau &= \epsilon^2\left(1 - \frac{\nu_{21}}{W}\right)t, \\ \xi &= \epsilon\left[x - \left(v_{\mathrm{g}} + \frac{\eta_2}{k} + \epsilon\nu_{32}\right)\right] \end{aligned} \tag{8}$$

where ν_i is the real constant depending on initial conditions and many parameters. Especially, focusing on the nonlinear and dissipation terms, we found the following points: (i) strong dissipation due to thermal conduction in dissipation coefficient is observed, and (ii) the interaction between wave nonlinearity and thermophysical property is clarified.

5. Summary

Weakly nonlinear propagation of shock waves in compressible water containing many microbubbles is theoretically investigated with a special focus on the thermal effects. As a result, thermal conduction strongly contributes the wave dissipation. Furthermore, the interaction between thermophysical property and wave nonlinearity is clarified.

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