

Comparisons about Stabilization Methods for Increasing Ultrasonic Vectorial Doppler Measurement Accuracy

超音波ベクトルドップラー計測を高精度化するための安定化法の比較

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1. Introduction

We have developed ultrasonic echo vectorial Doppler methods such as the multidimensional autocorrelation method [1,2] and multidimensional cross-spectral gradient method [3,4], etc. The methods have been used for measuring and imaging human soft tissue motion and blood flow, etc. Since the methods achieve accuracy that allows differentiating the measurements such as displacements and velocities, the strain and strain rate tensors are also observed. Simultaneously, we have developed the beamforming methods dedicated for performing the vectorial measurements such as the lateral modulation method (LM) achieved by crossing plural beams [1,2,5] and the spectral division method (SFDM) [6]. The methods allow performing inverse analyses of mechanical properties such as a shear modulus, etc [7].

To increase the measurement stability, we have been developing the over-determined (OD) systems using the LM and SFDM (e.g., [8,9]) with optimization methods such as the regularization method (e.g., [10]) and the maximum a posteriori (MAP) method [11], which has been increasing the measurement accuracy. In Ref. [10], the regularization is performed using the L2-norms about the differentiations of displacement vector components (L_2d), which is more effective than using those of vector components themselves (L_2i), whereas in Ref. [11], the L_2i is used in the MAP. Actually, we performed the MAP even if the displacement components were larger than the sampling intervals by using the Fourier phase matching method [11].

In this report, we perform the regularization with the Fourier phase matching method and the MAP with L_2d . The measurement accuracy and stability are also compared with the maximum likelihood method and the least-squares (LS) method.

2. Methods

When the OD system comprising Doppler equations is expressed as

$$\mathbf{F}\mathbf{d} = \boldsymbol{\theta}', \quad (1)$$

where $\mathbf{F}$ expresses a matrix of frequencies; and $\mathbf{d}$ and $\boldsymbol{\theta}'$ respectively express the target displacement vectors and the phase differences θ' calculated via the Fourier phase matching, the ML estimate $\mathbf{d}$ is obtained by solving

$$\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F}\mathbf{d} = \mathbf{F}^T\mathbf{C}^{-1}\boldsymbol{\theta}', \quad (2)$$

where $\mathbf{C}$ is the covariance matrix of $\boldsymbol{\theta}'$ and T denotes the transpose of a matrix. The covariance matrix $\mathbf{C}$ is estimated by ensemble or arithmetic averaging under the assumption of a locally stationary process.

And, the MAP estimate of $\mathbf{d}$ with L_2i is obtained by solving

$$(\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F} + \beta\mathbf{C}_d^{-1})\mathbf{d} = (\mathbf{F}^T\mathbf{C}^{-1}\boldsymbol{\theta}' + \beta\mathbf{C}_d^{-1}\mathbf{E}[\mathbf{d}]), \quad (3)$$

where $\mathbf{C}$ is the same as that in Eq. (2); and $\mathbf{C}_d$ and $\mathbf{E}[\mathbf{d}]$ are respectively the covariance matrix and the expectation about the target displacement vectors $\mathbf{d}$, which are *a priori* estimated by ensemble or arithmetic averaging under the assumption of a locally stationary process for *a priori* measurements $\mathbf{d}$ obtained by solving Eq. (1) or (2). When $\beta = 1$, Eq. (3) is a conventional equation, whereas when $\beta \neq 1$, the equation is controlled about *a priori* data. Alternatively, the MAP estimate of $\mathbf{d}$ with L_2d is obtained by solving

$$(\mathbf{F}^T\mathbf{C}^{-1}\mathbf{F} + \beta\mathbf{D}^T\mathbf{C}_{dd}^{-1}\mathbf{D})\mathbf{d} = (\mathbf{F}^T\mathbf{C}^{-1}\boldsymbol{\theta}' + \beta\mathbf{D}^T\mathbf{C}_{dd}^{-1}\mathbf{DE}[\mathbf{d}]), \quad (4)$$

where $\mathbf{D}$ is a Laplacian operator; $\mathbf{C}_{dd}$ is the covariance matrix about $\mathbf{Dd}$; and β is the same as that in Eq. (3).

In the least-squares and regularization senses which the author developed, highly confidential equations and important penalty terms are largely weighted, respectively. The weights are also determined and/or directional-dependently using variances about the target displacements *a priori* [12] or *a posteriori* [10]. In this report, for comparison, the regularizations were performed as follows:

$$(\mathbf{F}^T\mathbf{F} + \alpha\mathbf{V})\mathbf{d} = \mathbf{F}^T\boldsymbol{\theta}' \quad (5)$$

or

$$(\mathbf{F}^T\mathbf{F} + \alpha\mathbf{D}^T\mathbf{VD})\mathbf{d} = \mathbf{F}^T\boldsymbol{\theta}', \quad (6)$$

where $\mathbf{V}$ is a matrix comprising of variances about $\mathbf{d}$. $\alpha\mathbf{V}$ is the regularization parameter.

3. Phantom experiments

The synthetic aperture echo data were as the same as those used in Ref. [11] obtained from an agar phantom [40 (axial) $\times$ 96 (lateral) $\times$ 40 (elevational)mm³] having a central circular cylindrical inclusion (diameter, 10 mm; depth, 19mm) with a shear modulus different from that of the surrounding region, 2.63 and 0.80×10^6 N/m² (relative shear modulus, 3.29). The phantom was manually compressed in the lateral direction. A linear-array-type transducer with a nominal frequency of 7.5MHz was used (Aloka LNR5539). A rectangular ROI of 13.7 (axial, x) $\times$ 13.2 (lateral, y) mm² was centered on the inclusion (depths from 12.2 to 25.9 mm). The phantom was manually compressed in the lateral direction. A linear-array-type transducer with a nominal frequency of 7.5MHz was used (Aloka LNR5539). A rectangular ROI of 13.7 (axial, x) $\times$ 13.2 (lateral, y)mm² was centered on the inclusion (depths from 12.2 to 25.9 mm). For the agar phantom, the two same OD system as those in Ref. [11] were generated using four crossed, steered beams (Case 1, ± 20 and ± 30 degrees) and using four quasi-waves generated using SFD for a nonsteered single-beam scan (Case 2).

Here, the 2D autocorrelation method [1,2] was used, and the measurement accuracy and stability were evaluated for MAPs with (1) L₂d and (2) L₂i, (3) ML, regularizations with (4) L₂d and (5) L₂i, and (6) LS. For MAPs of (1) and (2), $\beta = 1$ and $\beta \neq 1$ were respectively used.

For MAPs and ML, the assumptions of independencies about $\mathbf{d}$ components and θ estimated from respective beams or quasi-beams were effective (specific omitted). Fig. 1 shows for Case 1 (a) means and (b) SNRs of measured lateral, axial and shear strains, and Fig. 2 shows for Case 2 SNRs of strains. In both cases, MAP and regularization with L₂d yielded more stable and high SNR measurements than others. However, specifically in Case 1 the normal MAP yielded much stable results, which lowered spatial resolutions substantially. Then, the resolution problem was significantly mitigated by using β less than 1, which increased SNR. In contrast, since Case 2 is originally ill-conditioned, a large window was used for the static estimation or β larger than 1 was successfully used. Summarizing, it was concluded that the regularization with L₂d was the best for the experimental data through considering the spatial resolution as well as the accuracy (images omitted).

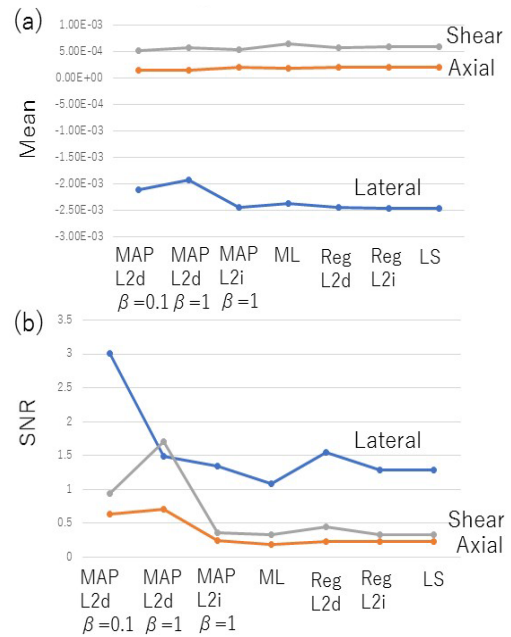


Fig. 1 In Case 1, (a) means and (b) SNRs of measured lateral, axial and shear strains.

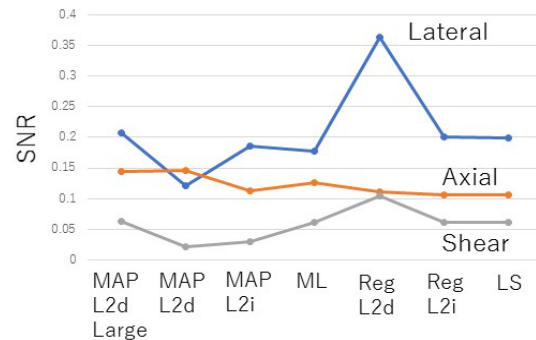


Fig. 2. SNRs in Case 2.

4. Conclusions

Proposed MAP and regularization with L₂d were effective for yielding stable and accuracy measurements. The methods will also be compared with a gradient and/or a conventional regularization next. The stabilizations will also be performed for the cross-spectral phase gradient method.

References

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