# Acoustic waves propagating through a solid-fluid superlattice by resonance with Lamb wave modes

ラム波モードとの共鳴により固体流体超格子を透過する音響波

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### 1. Introduction

Frequency gaps caused by periodic structures in phononic crystals (or superlattices) are useful for creating phonon filters, phonon mirrors and vibration insulation devices in selective frequency range. In addition, recently, phononic crystals with locally resonant structural units has attracted much attention [1, 2], because these exhibit frequency gaps with lattice constants considerably smaller than the relevant sonic wavelength. One-dimensional superlattices consisting of solid and fluid layers are simple examples of such sonic crystals [3, 4]. In particular, the frequency gap due to local resonance has been investigated theoretically.

Recently, we theoretically investigated the spectrum transmission of acoustic waves transmitted through a plate in a fluid. When an acoustic wave is vertically incident on the interface, the transmission spectrum has Lorentz-type symmetric resonance peaks around the eigenfrequencies of the longitudinal wave confined in the solid plate. In addition, we show that the Fano-type asymmetric resonance peaks [5] appear near the eigenfrequencies of the transverse wave when the acoustic wave is obliquely incident at an angle near vertical, and their shapes were theoretically investigated [6].

In the present study, the transmission spectra of acoustic waves obliquely incident on a solid-liquid superlattice are theoretically examined for a wide range of incident angles. In this case, the mixing of the longitudinal wave component and the transverse wave component in the solid layer becomes large, and Lamb waves are generated. The relation between the resonance peaks in the transmission spectrum and the eigenfrequencies of the Lamb waves is theoretically examined. Moreover, the relation between the transmission zeros due to local resonance and the frequency gaps is discussed based on the singularity of the generalized Bloch wavenumber.

## 2. Theoretical Method

The transmission spectrum is calculated based on the transfer matrix method. We solve the Euler equation and the elastic equation for the liquid layer and the solid layer, respectively, and obtain the velocity and stress fields in each layer. The boundary conditions to be imposed are that the shear stresses at the interface are zero and the velocity and stress components perpendicular to the interfaces are continuous at the interface.

## 3. Numerical results and discussions

As a first example, we show in Fig. 1 the transmittance of an acoustic waves propagating through an Al layer in water as a function of angle of incidence and frequency. For reference, we show in Fig. 2 the eigenfrequencies of Lamb waves in an Al layer placed in a vacuum as a function of frequency and propagation angle. The frequencies that give transmission peaks are in good agreement with the eigenfrequencies of the Lamb waves, even though the boundary conditions used to derive the Lamb wave and the those used in the present system are different. The boundary conditions for Lamb waves are that, in addition to shear stress, normal stresses are 0 on the solid surface.

As an example of a periodic multi-layer structure, we show in Fig. 3 the transmittance calculated for acoustic waves propagating through an 8-period solid-fluid superlattice consisting of

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alternating stacks of Al and water layers. Figure 3 shows a band structure that reflects the Lamb wave dispersion relations. In the transmission spectrum of the multi-period structure, each transmission peak is divided into eight (the number of solid layers), and the tails overlap to form a band. In the band structure, there are two types of band gaps, namely, the band gap due to the periodic structure and the one due to local resonance.



**Fig. 1** Transmittance of acoustic waves propagating through a single Al layer in water. Here, the frequency  $\omega$  is normalized using  $\omega_1 = \pi c_1 / d_s$ , where  $d_s$  is the thickness of the Al layer and  $c_1$  is the sound velocity of the longitudinal acoustic waves in the Al layer.



**Fig. 2** Eigenfrequencies of the Lamb waves as functions of propagation angle and frequency. The red and blue lines are for the symmetric and antisymmetric Lamb waves, respectively.

It was shown that the resonance peaks in the transmittance of acoustic waves propagating through an Al layer in water agree with the eigenfrequencies of the Lamb waves for all incident angles and frequency regions. The eigenfrequencies of Lamb waves are determined by the stress free boundary conditions on both surfaces of the solid layer, which is different from the boundary condition imposed for the present system. In the presentation, we will theoretically explain the relationship between the resonance peaks in transmission and the eigenfrequencies of Lamb waves in more detail. Furthermore, the relationship between the transmission zeros seen in the single solid layer system and the frequency gap due to local resonance will discussed in terms of the singularity of the generalized Bloch wavenumber.



**Fig. 3** Transmittance of acoustic waves propagating through an Al/water superlattice. The number of periods is assumed to be 8, and the thickness of an Al layer  $d_s$  is assumed to be  $d_s=2d_f$ , where  $d_f$  is the thickness of a water layer.

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#### References

- Z. Liu, X. Zhang, Y. Mao, Y. Y. Zhu, Z. Yang, C. T. Chan, and P. Sheng, Science 289(5485), 1734, 213 (2000).
- N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun and X. Zhang, Nat. Mater. 5, 452 (2006).
- 3. S. Mizuno, Jpn. J. Appl. Phys. 55, 017302 (2016).
- 4. I. Quotane, E. H. El Boudouti, and D-R. Bahram, Phys. Rev. B. **97**, 024304 (2018).
- 5. U. Fano, Phys. Rev. 124, 1866 (1961).
- 6. S. Mizuno, J. J. Appl. Phys. 59, SKKA02 (2020).