3J1-1

Photoacoustic Beacon Positioning with Kalman Filter

カルマンフィルタを用いた光音響ビーコン位置検出

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1. Introduction

Medical modalities using photoacoustically generated ultrasound (US) has been drawing much attention. The advantages of photoacastics (PA) is the high spatial resolution [1] and capability of miniaturizing US transmitters. By using such small transmitters, we are developing an acoustically detectable interventional system called the PA beacon system (**Fig. 1**) [2], which will emit US from the therapeutic device such as a guidewire and extracorporial US probes detects the device position. We believe such a system will contribute to secure and high thoughput echo-guided intervention.

For the PA becon system to be practical, it is essential to detect the 3D position of the device. Though positioning using a 2D array probe would be a candidate, the use of a common 1D probe would be prefereble for the becon to be widely used. **Figure 2(a)** shows the geometry of a 1D probe and a beacon. Since a 1D probe has many acoustic elements along the *x*-axis, the *x* beacon position can be geometrically obtained with time of flight. In *y-z* plane, however, the probe has only one element and cannot detect the beacon position appropriately. To solve this problem, we previously proposed a 3D beacon positioning with a probe elevation (*y*-axis) directivity [3]. However, it was not robust under low signal-to-noise ratio (SNR).

The objective of this study was to increase the robustness of beacon positioning in an elevation direction and propose an elevation-positioning method using the extended Kalman filter (EKF).

2. Method of beacon positioning

2.1. Basic theory of PA beacon positioning

For simplicity, a beacon is assumed to be in the *y-z* plane, as shown in Fig. 2. The signal propagation in the *y-z* plane can be treated as that of a point signal source and rectangle receiver. Sound pressure waves on a rectangle element $p(t)$, which are convolutions of the pressure generated by the vibration speed of a point source $u(t)$, is expressed as Eq. (1) [4].

$$p(t) \propto \frac{u\left(t - \frac{r - w\sin\theta}{c}\right) - u\left(t - \frac{r + w\sin\theta}{c}\right)}{r\sin\theta} \qquad (1)$$

Assuming $\theta \neq 0$, the convolution of the received pulses splits into two pluses, as shown in Fig. 2(b). The condition for a split signal is obtained from Eq. (1) as follows:

$$\underbrace{r - w\sin\theta}_{\stackrel{\mathrm{def}}{=} L_\mathrm{s}} \le t \cdot c \le \underbrace{r + w\sin\theta}_{\stackrel{\mathrm{def}}{=} L_\mathrm{e}}, \qquad (2)$$

where c denotes the speed of sound. A beacon

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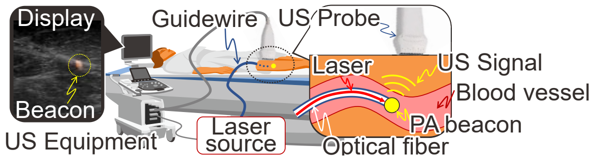
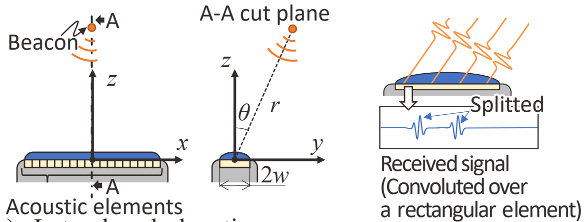


Fig. 1 PA beacon system



(a) Lateral and elevation geometry of 1D array probe.

(b) Split signal.

Fig. 2 Geometry of US probe and PA beacon

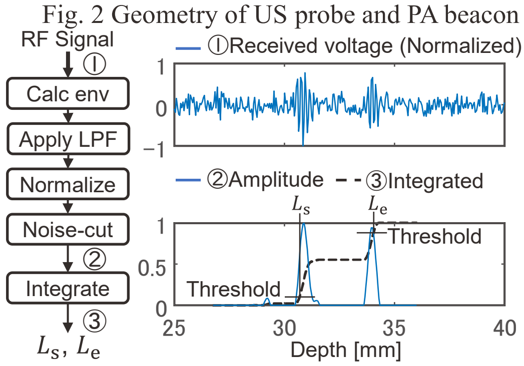


Fig. 3 Positioning process and processed signals

position can be obtained with L_s and L_e, which are defined in Eq. (2), as

$$r = (L_\mathrm{e} + L_\mathrm{s})/2., \quad \sin\theta = (L_\mathrm{e} - L_\mathrm{s})/(2w). \qquad (3)$$

Therefore, we can detect a beacon position by detecting L_s and L_e from recieved signals with the following procedure and as illustrated in **Fig. 3**.

Step 1: Computing an envelope of the signal.
Step 2: Appling low pass filter to reduce noise.
Step 3: Normalizing the filterd signal.
Step 4: Cutting the noise floor and normalizing.
Step 5: Integrating the amplitude.
Step 6: Detecting L_s and L_e at which the integration exceeds a threthold.

2.2. Kalman filter design

The EKF is composed of two steps: model-based prediction and observation-based correction. In the prediction step, the EKF estimates the current state of the system based on a linearized equation: $\boldsymbol{x}_{k+1} = \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{v}_k$, where $\boldsymbol{x}_k$, $\boldsymbol{v}_k$, and k represents the state, the process noise, and the time step, respectively. In the correction step, the EKF estimates the true state based on an observation equation: $\boldsymbol{y}_k = \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{w}_k$, where $\boldsymbol{y}_k$ and $\boldsymbol{w}_k$ represents the observation and the observation noise, respectively. This section explains the EKF design to reduce the beacon-positioning error in the *y*-*z* plane.

The state of a beacon-positioning system can be determined by the position and speed of a beacon. Since, these cannot be measured directly and are calculated from a beacon signal by using Eq. (3), and a backward difference (e.g. $\Delta y_{\mathrm{B},k} \simeq y_{\mathrm{B},k} - y_{\mathrm{B},k-1}$). To refer measured values in EKF process, we designed the state $\boldsymbol{x}_k$, expressed by Eq. (4), including L_s and L_e, which can be measured. The observation equation is $\boldsymbol{y}_k = \boldsymbol{x}_k$ and $\boldsymbol{C}_k = \boldsymbol{E}$.

$$\boldsymbol{x}_k = \begin{bmatrix} L_{\mathrm{e},k} & L_{\mathrm{s},k} & y_{\mathrm{B},k} & z_{\mathrm{B},k} & \Delta y_{\mathrm{B},k} & \Delta z_{\mathrm{B},k} \end{bmatrix}^T \tag{4}$$

The time derivative of $\boldsymbol{x}_k$ results in the following prediction equation where $\hat{\boldsymbol{x}}_{k+1}^-$ denotes a *priori* estimation for the $k+1$ time step.

$$\hat{\boldsymbol{x}}_{k+1}^- = \underbrace{\left[\begin{array}{c|cc} \boldsymbol{E}_4 & (y_{\mathrm{B},k}+w)/L_{\mathrm{e},k} & z_{\mathrm{B},k}/L_{\mathrm{e},k} \\ & (y_{\mathrm{B},k}-w)/L_{\mathrm{s},k} & z_{\mathrm{B},k}/L_{\mathrm{s},k} \\ \hline & \multicolumn{2}{c}{\boldsymbol{E}_2} \\ \hline \boldsymbol{0}_{2\times 4} & \multicolumn{2}{c}{\boldsymbol{E}_2} \end{array}\right]}_{\overset{\mathrm{def}}{=} \boldsymbol{A}_k} \hat{\boldsymbol{x}}_k \tag{5}$$

Now we can obtain a *posteriori* estimation for the $k+1$ step: $\hat{\boldsymbol{x}}_{k+1}$ with the following equations:

$$\boldsymbol{P}_{k+1}^- = \boldsymbol{A}_{k+1}\boldsymbol{P}_{k+1}\boldsymbol{A}_{k+1}^T + \boldsymbol{V}_{k+1}\,, \tag{6}$$

$$\boldsymbol{g}_{k+1} = \boldsymbol{P}_{k+1}^-\left(\boldsymbol{P}_{k+1}^- + \boldsymbol{W}_{k+1}\right)^{-1}, \tag{7}$$

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1}^- + \boldsymbol{g}_{k+1}\left(\boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1}^-\right), \tag{8}$$

$$\boldsymbol{P}_{k+1} = \left(\boldsymbol{E} - \boldsymbol{g}_{k+1}\right)\boldsymbol{P}_{k+1}^-\,, \tag{9}$$

where P_k and P_k^- are the estimation-error covariances of $\hat{x}_k$ and $\hat{x}_k^-$, $\boldsymbol{g}$ denotes a Kalman gain, and $\boldsymbol{V}_k$ and $\boldsymbol{W}_k$ are the drive and measurement error covariances, respectively. These can be designed based on the expected measurement error and probe motion.

3. Simulation method

This section describes the numerical simulation to validate the proposed elevation-positioning method. The simulation consists of an acoustic part, which is implemented in MATLAB® (MathWorks, Inc.) with FieldII library [5, 6], and a filterling part in MATLAB®.

The geometrical setup of the acoustic part is shown in **Fig. 4**, i.e., it illustrates the setting of the US transmitter (a rectangle of 0.3 mm×0.3 mm), a reciever that immitates the L11-5V linear probe (Verasonics, Inc.), and the scattering particle. The simulation was conducted assuming the following situation; the particle scatters the impulse signal from the transmitter, then the reciever recieves the scattered signal. The transmitter transmits an US pulse when $t = -z_\mathrm{B}/c$ so that a particle reflects the pulse when $t = 0$. The sample rate was set to 31.25 MHz and white Gaussian noise was added to the obtained signals so that the SNR would be 20 dB.

In the filtering part, the proposed positioning method processes the obtaind acoustic simulation results. The filtering rate was 20 Hz.

4. Simulation results

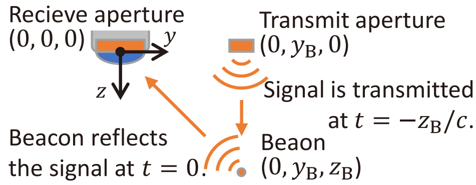
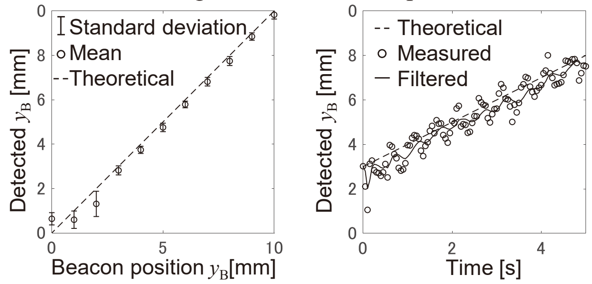


Fig. 4 Simulation setup



(a) When beacon is stable (b) When beacon moves

Fig. 5 Simulation results

Figure 5(a) shows the positioning accuracy of a beacon that does not move. The dots and error bars denote the mean and standard derivitive of 1024 samples, respectivly. The root mean square error (RMSE) was 0.8 mm ($0\ \mathrm{mm} \leq y_\mathrm{B} < 3\ \mathrm{mm}$) and 0.3 mm ($3\ \mathrm{mm} \leq y_\mathrm{B}$).

Figure 5(b) shows the results when a beacon moves in $y_\mathrm{B} = t + 0.5\sin\pi t$, which imitates bluring induced by the operator when measuring. The unfiltered and filterd detection error were -0.3 (0.5) mm and -0.3 (0.2) mm, respectively. Each value represents the mean (standard derivitive).

5. Conclusion

We proposed an elevation-poisitioning method for reducing the positioning errors of a PA beacon. The simulation showed that the proposed method provides good estimation (RMSE 0.3 mm) when $y_\mathrm{B} \geq 3\ \mathrm{mm}$. However, the error was large (RMSE 0.8 mm) when $y_\mathrm{B} < 3\ \mathrm{mm}$. This might be due to violation of the assumption: $\theta \neq 0$. The simulation also showed that the proposed method reduced errors induced by hand blur and that beacon positioning with a sensor-fusion system will be able to use US signals and those of additional sensors (e.g. US probe position sensors).

References

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