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Energy Trapping of Circumferential SH Wave at a Groove in a Pipe

パイプ中溝部における周方向 SH 波のエネルギトラップ

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1. Introduction

Local changes in the shape and material of the elastic body can confine the vibration to that area. For example, the energy trapping of thickness vibration is used in a Quartz Crystal Microbalance (QCM) [1-6]. In a QCM, which is generally a circular quartz plate with gold electrodes at the center of both surfaces, the vibration energy is confined in a thicker electrode area. Because the resonance is not affected by the geometry and supports located away from the electrodes, it becomes very stable. Using this prominent characteristic, the QCM is used as a highly sensitive sensor for detecting small variations in mass caused by oxide growth, adhering viruses and proteins, polymers, and molecules.

Such energy trapping has been observed not only in flat plates, but also in round bars and pipes. For example, Johnson et al. [7] presented energy trapping of a circumferential surface wave in a stepped region in a circular bar. Hayashi et al. [8-11] numerically and experimentally verified that circumferential propagation modes, which resonate in the circumferential direction of the pipe, cause energy trapping due to the change in pipe thickness.

Considering the energy trapping in a pipe with a groove, the energy trapping area can be very thin in a pipe with a groove, we can expect to realize highly sensitive sensors. Moreover, electrodes are not required on the vibration surface, the response characteristics can be improved. However, Hayashi et al. [11] presented and discussed the theoretical and experimental results of out-of-plane vibrations, and little was said about the energy trap for the modes of in-plane vibrations. For example, considering the application of this phenomenon to biosensors, it is known that the Q-value of out-of-plane vibrations decreases due to leakage into the solution when the waves are directed out-of-plane, while the Q-value of SH waves does not decrease due to leakage into the fluid.

In this study, the energy trapping of SH waves propagating in a pipe with a groove is investigated.

2. Energy Trapping of SH Waves Predicted by Dispersion Curves

Figure 1 shows the dispersion curves for aluminum alloy pipes with a longitudinal wave velocity $c_L = 6260\ \mathrm{m/s}$ and a transverse wave velocity $c_T = 3000\ \mathrm{m/s}$ in which the axial vibration is dominant and the guided wave mode is represented by the axial wavenumber. The figure shows only the modes with the circumferential order $n = 5$. n is the circumferential mode order, meaning that it has a circumferential distribution of $\cos(n\theta)$. k is the wavenumber of the guided wave propagating in the longitudinal (z) direction. The vertical axis is frequency, and the horizontal axes on the right- are the real part of wavenumber k, $\mathrm{Re}(k)$, and left-hand sides are the imaginary part of wavenumber k, $\mathrm{Im}(k)$. We compare the dispersion curves of three pipes with different diameters and thicknesses in order to predict whether energy trapping occurs when a groove is placed on the outer surface or the inner surface of the pipe. The black line are the dispersion curves for a pipe with an outer diameter of 8 mm and a thickness of 0.4 mm, the blue line are the dispersion curves for a with an outer diameter of 7.8 mm and a pipe thickness of 0.3 mm, and the red line are the dispersion curves for a pipe with an outer diameter of 8 mm and a pipe thickness of 0.3 mm, where the solid line is the real part of wavenumber k and the dashed line is the imaginary part of wavenumber k. The frequency for $k = 0$ is called the cutoff frequency, at which the mode does not propagate in the longitudinal direction and has the distribution of $\cos(5\theta)$ with the circumferential resonance.

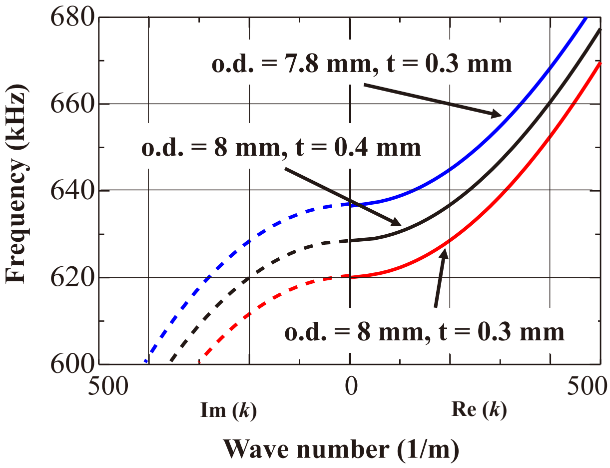

Fig.1 Dispersion curves for pipes of different diameters and thicknesses

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In the case of a pipe with the diameter 8 mm and 0.4 mm thickness and a pipe with the diameter 8 mm and 0.3 mm thickness, the cutoff frequency of the diameter 8 mm and 0.3 mm thick pipe is 620 kHz and the wavenumber of the diameter 8 mm and 0.4 mm thick pipe at that frequency is purely imaginary, as $k = 200.2i$. When a pipe groove is excited at the cutoff frequency, a standing wave with the distribution $\cos(5\theta)$ in the circumferential direction is formed without propagation in the longitudinal direction as $k = 0$ at the groove. At the same time, outside the groove, the longitudinal displacement decreases exponentially with a distribution of exp(-200.2z), since $k = 200.2i$. This means that the energy is trapped in the pipe groove while resonating in the circumferential direction.

On the other hand, in the case of a pipe with the diameter of 8 mm and 0.4 mm thickness and a pipe with the diameter of 7.8 mm and 0.3 mm thickness, the cutoff frequency of the diameter of 7.8 mm and 0.3 mm thickness is 636.5 kHz, and the wavenumber of the diameter of 8 mm and 0.4 mm thick pipe at that frequency is $k = 196.5$ (real). This means that wave propagates toward the outside of the groove and there is no energy trapping in the pipe grooves.

3. Energy Trapping Mode of SH Waves Calculated by Semi-Analytical Finite Element Method

Because the above-mentioned analysis uses dispersion curves for a pipe of an infinite length, the energy trapping frequencies at a circumferential groove with a finite length cannot be predicted precisely. Therefore, the energy trapping frequencies are derived using a semi-analytical finite element (SAFE) calculation [12]. Figure 2 shows one of the resonant modes obtained as an eigenvalue in the SAFE, in which the vibration in the pipe axial direction are concentrated in a groove. The displacement in the axial direction of the pipe at a given moment is shown in color. These vibration appear as standing waves in the grooves, with the blue and red parts being the belly and the yellow-green between them being the nodes. The calculated resonant frequency of 623 kHz is between the cutoff frequency of the diameter 8 mm and 0.4 mm thickness and the diameter 8 mm and 0.3 mm thickness, as predicted by the analysis of the dispersion curves in Figure 1. The fact that the energy trapping can be obtained even for SH wave vibration implies that this phenomenon offers promise for a wide variety of applications.

4. Summary

In this study, the energy trapping of circumferential SH waves in the pipe grooves is investigated. After predicting the energy trapping in the pipe groove using dispersion curves, the resonance frequencies and vibration modes were calculated numerically using the SAFE. The frequency is close to that expected by the dispersion curve. In addition, the energy trapping of even SH-wave vibrations is expected to increase the sensitivity when applied to devices such as sensors.

The energy trapping modes will mainly contribute to improvement in the sensitivities and responsiveness of various sensors using vibrations, as in QCMs. In a QCM, the energy trapping occurs in the thick electrode area, which limits their sensitivity. On the contrary, the energy trapping area can be very thin in a pipe with a groove. The sensitivity of the sensor is expected to be improved by the increase in the adsorbed mass. Moreover, the in-plane modes can be generated and detected without contact and without electrodes by using electromagnetic acoustic transducers or laser ultrasonics, thereby resulting in sensors with extremely high sensitivities.

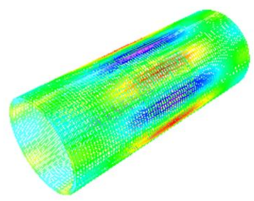

Fig.2 Energy trapping mode of SH wave
Color indicates the displacement in the pipe axial direction

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