Effects of target scatterer size on ultrasonic sound velocity estimation based on delay time distribution

超音波の遅延時間分布を用いた音速推定法における対象散乱 体径の影響に関する検討

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1. Introduction

Medical ultrasound is useful for the diagnosis of diseases and the observation of various organs. However, the spatial resolution and the image quality of ultrasound images are deteriorated by the sound velocity distribution in *in vivo*. Thus, estimating the sound velocity distribution is expected to solve this problem.

We have developed the sound velocity estimation method utilizing the reception time of scattered waves at each element in an ultrasonic probe¹). We estimated the sound velocity of water by changing the diameter of the scatterers. As a result, an increase in the estimation error was confirmed as the scatterer size became larger²). In the present study, the effect of the target scatterer size on the sound velocity estimation was examined by basic and simulated experiments in detail.

2. Principles and methods

2.1 Sound velocity estimation method¹⁾

It is assumed that the ideal point scatterer exists at a depth d below the central element of the linear probe as the target as shown in **Fig. 1(a)**. The scattered waves from the point scatterer are received by each element in the probe. $T(x_k)$ is the theoretical value of the propagation time of the scattered wave from the point scatterer to the position $(x_k, 0)$ of the element k. It is given by Eq. (1).

$$T(x_k) = \frac{\sqrt{x_k^2 + d^2}}{c} \,. \tag{1}$$

The square of the measured propagation time $t(x_k)$ of the scattered wave from the point scatterer to the position $(x_k, 0)$ of the element k is matched with the square of $T(x_k)$.

$$\{T(x_k)\}^2 = \frac{1}{c^2}x_k^2 + \frac{d^2}{c^2} = ax_k^2 + b, \qquad (2)$$

where c is the sound velocity. From Eq. (2), the sound velocity c and depth d are estimated as follows.

$$\hat{c} = \sqrt{\frac{1}{a}} , \hat{d} = \sqrt{\frac{b}{a}}.$$
(3)

2.2 Simulation of propagation of scattered wave when the scatterer has a size

The scattering of ultrasonic waves on the surface of a scatterer having a finite size is equivalent to the scatterings from many point scatterers on the



Fig. 1 Schematic diagram of reception of scattered waves. (a) An optimum point scatterer. (b) Scatterer which has a size. surface of the scatterer.

In this simulation, we calculated the received waveform $h_k(t)$ at k-th element from the scatterer having a size. The calculation model is shown in Fig. 1(b). The diameter of the scatterer is 2r. The 2M+1point scatterers are set on the surface of the scatterer at Δl intervals. The 0th point scatterer is set under the central element. The plane wave is incident on the scatterer by assuming that the focused wave can be approximated by a plane wave in the focal region. x'_m and y'_m show the lateral and vertical distances, respectively, from the 0th point scatterer to *m*-th point scatterer.

 $T(x'_m, x_k)$ shows the ultrasonic propagation time of scattered wave which is transmitted from the element above the point scatterer *m*, scattered by the *m*-th point scatterer, and received by the element *k*. It is given by Eq. (4).

$$T(x'_m, x_k) = \frac{d + y'_m + \sqrt{(x_k - x'_m)^2 + (d + y'_m)^2}}{c}.$$
 (4)

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The delay time τ_{mk} of $T(x'_m, x_k)$ from $T(x'_0, x_k)$ is obtained by

 $\tau_{mk} = T(x'_m, x_k) - T(x'_0, x_k).$ (5) Then, the simulated received waveform $h_k(t)$ is determined by Eq. (6).

$$h_k(t) = \sum_{m=-M}^{M} h(t - \tau_{mk}),$$
 (6)

where $h(t - \tau_{mk})$ is the signal from the *m*-th point scatterer received by the element *k*. The measured waveform from a tungsten wire with a diameter of 0.03 mm at the central element was used as h(t).

2.3 Experimental method

The effect of the target scatterer size on the sound velocity estimation was confirmed by the basic experiment using scatterers with different sizes. Ultrasonic diagnosis equipment (Prosound α 10, Hitachi Aloka) was used with a linear probe (UST-5412, Hitachi Aloka). The transmitting frequency was set at 7.5 MHz. The sampling frequency was set at 40 MHz. 96 elements were used for transmitting and receiving ultrasonic beams.

A tungsten wire with a diameter of 0.03 mm, nylon wires with diameters of 0.25, 0.50, 0.75, 0.90 mm, and silicone rubber wires with diameters of 2.0, 3.0, 4.0, 5.0, 6.0 mm were used as the scatterers. The top surface of each wire was set at the depth of 30 mm from the probe surface in the water. We measured 10 times for each wire.

We determined the ultrasonic propagation time $t(x_k)$ by detecting the peak of the received waveform at each receive element. Figure 2(a) shows the received waveform at the central element of the scattered wave from the scatterer with a diameter of 0.25 mm. Figure 2(b) shows the simulated received waveforms of the scattered waves from the anterior and posterior walls of the scatterer with a diameter of 0.25 mm. We confirmed that the scattered waves from the anterior wall of the scatterer and posterior wall of the scatterer interfered in Fig. 2(a).

From Fig. 2(b), we determined the position of the peak (O) where the scattered waves from the anterior wall and that from the posterior wall do not interfere with each other. For a diameter of $2r \ge 0.25$ mm, the peak positions were similarly determined. For a diameter of 2r = 0.03 mm, the delay time between the scattered waves from the anterior and posterior walls was approximately only 0.1 period of the transmitted wave. Therefore, the position of the peak was determined in the interfered waveform for the wire with 2r = 0.03 mm.

We conducted simulated experiments under the same measurement conditions as the basic experiment.

3. Results and discussion

The true sound velocities in the experiments were 1495 m/s (*r*=0.90~6.0 mm), 1496 m/s (*r*=0.50, 0.75 mm), and 1497 m/s (*r*=0.03, 0.25 mm) from the temperature of water. The sound velocities estimated in the basic and simulated experiments are shown by red and blue solid circles in **Fig. 3**, respectively. We confirmed that the estimation error between the estimated and true values of the sound velocity increases as the diameter of the scatterer becomes large in both basic and simulated experiments. The sound velocity estimation results of basic and simulated experiments were almost the same.

We can explain the reason why the estimation error increases as the diameter of the scatterer becomes large using Fig. 1(b) as follows. The propagation path of the scattered wave from the 0th point scatterer is shown with the blue line in Fig. 1(b), and the propagation path of the scattered wave from the *m*-th point scatterer is shown by the red line in Fig. 1(b). The 0th point scatterer under the central element exists regardless of the size of the scatterer. At the central element, the scattered wave from the 0th point scatterer is received earliest. However, the element k receives the scattered wave from the m-th point scatterer earliest. As a result, the curvature a in Eq. (1) becomes smaller as the red parabola in Fig. 1(b). Therefore, the estimated value of the sound velocity calculated from the curvature a as in Eq. (3) becomes faster and the estimation error increases as the curvature a in Eq. (2) becomes smaller.



4. Conclusion

In the present paper, we investigated the effect of the scatterer diameter on the sound velocity estimation through basic and simulated experiments. We will develop a method for estimating the sound velocity considering the effect of scatterer diameter. References

- 1. K. Abe, et al., J. Med. Ultrason. 46, pp. 27-33
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 2. K. Abe, *et al.*, Jpn. J. Med. Ultrason. 46, Suppl S 573 (2019)