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Basic Theory on Ultrasound Propagation in a Liquid Containing Encapsulated Bubbles toward Medical Application

弾性膜で覆われた気泡を含む液体中の超音波伝播の医療応用に向けた基礎理論創成

Yusei Kikuchi[1‡] and Tetsuya Kanagawa[2] ([1]Grad. Sch. Syst. Inf. Eng., Univ. of Tsukuba; [2]Fac. Eng., Inf. & Syst., Univ. of Tsukuba)
菊地勇成 [1‡], 金川哲也 [2] ([1]筑波大院シス情, [2]筑波大シス情)

1. Introduction

In recent years, an application of ultrasound in the medical field such as therapy and diagnosis has been developed. The use of microbubble is drastically improved and enhanced a resolution of the image and an efficiency of therapy. DDS (drug delivery system) [1] and UCA (ultrasound contrast agent) [2] are the famous and expected examples for application. From the physical viewpoint, the precise understanding of the interaction between ultrasound propagation and microbubble oscillations has then been desired.

The microbubble used in such applications is usually covered by a lipid shell, membrane, and so on. Church [3] and Hoff et al. [4]. established the theory of oscillations of an encapsulated bubble (i.e., bubble covered by an elastic shell) and derived the equation of motion for radial oscillations. However, these theories were restricted to the single bubble or some bubbles. In this paper, we theoretically studied nonlinear propagation of ultrasound in liquids containing many encapsulated microbubbles to extend the previous theory for a single bubble [3, 4] into that for a number of bubbles. Based on the singular perturbation method and Hoff's model [4], we succeeded the derivation of an effective wave equation for ultrasound propagation in liquids containing many encapsulated bubbles.

2. Formulation of the problem

Let us consider the weakly nonlinear (i.e., finite but small amplitude) propagation of plane progressive pressure (or ultrasonic) waves radiated from a sound source placed at the bubbly liquid.

At an initially quiescent state, all the dependent variables including the bubble radius and the void fraction are uniform. To focus on the effect of shell of encapsulated bubble, we impose some assumptions: the bubbles do not coalesce, break up, appear, and disappear. The bubbles are spherical, and these oscillations are spherically symmetric. In

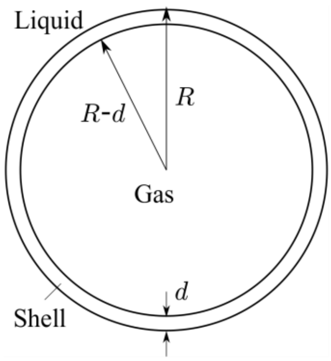

Fig. 1 Conceptual illustration of an encapsulated bubble.

addition, the liquid compressibility, the viscosity of gas inside the bubbles and the thermal conductivities of both phases are neglected. These assumptions are almost the same as those in our previous studies [5, 6]. In this paper, the Church [3] or Hoff model [4], i.e., bubble dynamics as oscillations of encapsulated bubble, is installed for the equation of motion, instead of the Rayleigh-Plesset (or Keller-Miksis) model. We than can investigate theoretically the effect of viscosity and rigidity of the encapsulated shell on wave propagation.

3. Basic equations for bubbly flows

The set of basic equations based on a two-fluid model [5] is used. Firstly, the conservation of mass and momentum for gas and liquid phases are

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*)+\frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^*u_{\mathrm{G}}^*)=0, \tag{1}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*]+\frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^*u_{\mathrm{L}}^*]=0, \tag{2}$$

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*u_{\mathrm{G}}^*)+\frac{\partial}{\partial x^*}\left(\alpha\rho_{\mathrm{G}}^*u_{\mathrm{G}}^{*2}\right)+\alpha\frac{\partial p_{\mathrm{G}}^*}{\partial x^*}=F^*, \tag{3}$$

$$\begin{aligned}\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*u_{\mathrm{L}}^*]&+\frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^*u_{\mathrm{L}}^{*2}\right]\\&+(1-\alpha)\frac{\partial p_{\mathrm{L}}^*}{\partial x^*}+P^*\frac{\partial\alpha}{\partial x^*}\\&=-F^*,\end{aligned} \tag{4}$$

where t^* is the time, x^* space coordinate normal

[1]kikuchi.yusei.sp@alumni.tsukuba.ac.jp
[2]kanagawa.tetsuya.fu@u.tsukuba.ac.jp

to the wave front, α void fraction $(0 < \alpha < 1)$, ρ^* density, u^* fluid velocity, p^* pressure, and F^* interfacial momentum transport; the subscripts G and L denote volume-averaged variables in the gas and liquid phases, respectively. In addition to the volume-averaged pressure p_{G}^* and p_{L}^*, the liquid pressure averaged on the bubble-liquid interface, P^*, is introduced.

We utilized Church-Hoff model [5, 6] (not Rayleigh-Plesset equation) as the equation of motion of the bubble dynamics to clarify the effect of viscosity and rigidity of the shell:

$$\rho_{\mathrm{L}}^*\left[R^*\frac{\mathrm{D}_{\mathrm{G}}^2R^*}{\mathrm{D}t^{*2}}+\frac{3}{2}\left(\frac{\mathrm{D}_{\mathrm{G}}R^*}{\mathrm{D}t^*}\right)^2\right] = p_{\mathrm{G}0}^*\left[\left(\frac{R_0^*}{R^*}\right)^{3\kappa}-1\right]-p_i^*(t) - \frac{4\mu_{\mathrm{L}}^*}{R^*}\frac{\mathrm{D}_{\mathrm{G}}R^*}{\mathrm{D}t^*}-12\mu_{\mathrm{S}}^*\frac{d_{\mathrm{S}0}^*R_0^{*2}}{R^{*4}}\frac{\mathrm{D}_{\mathrm{G}}R^*}{\mathrm{D}t^*} - 12G_{\mathrm{S}}^*\frac{d_{\mathrm{S}0}^*R_0^{*2}}{R^{*3}}\left(1-\frac{R_0^*}{R^*}\right), \qquad (5)$$

where R^* is the bubble radius, G_{S}^* shear modulus, μ_{S}^* shear viscosity, $p_{\mathrm{G}0}^*$ initial gas pressure, $p_i^*(t)$ driving pressure of liquid, μ_{L}^* liquid viscosity, $d_{\mathrm{S}0}^*$ initial shell thickness, κ ratio of specific heats of gas, and R_0^* initial bubble radius.

4. Nondimensionalization

Independent variables are nondimensionalized

$$t=\frac{t^*}{T^*},\qquad x=\frac{x^*}{L^*}, \qquad (6)$$

where T^* and L^* are a typical period and wavelength of the wave concerned, respectively.

Furthermore, we nondimensionalized some parameters and evaluate the size of the nondimensionalized ratios:

$$\frac{d_{\mathrm{S}0}^*}{R_0^*}=d_{\mathrm{S}0}\epsilon, \qquad (7)$$

$$\frac{\mu_{\mathrm{S}}^*}{\rho_{\mathrm{L}0}^*U^{*2}T^*}=\mu_{\mathrm{S}}, \qquad (8)$$

$$\frac{G_{\mathrm{S}}^*}{\rho_{\mathrm{L}0}^*U^{*2}}=G_{\mathrm{S}}, \qquad (9)$$

$$\frac{\mu_L^*}{\rho_{\mathrm{L}0}^*U^{*2}T^*}=\mu_{\mathrm{L}}\epsilon, \qquad (10)$$

$$\frac{R_0^*}{L^*}=\Delta\sqrt{\epsilon}, \qquad (11)$$

where $d_{\mathrm{S}0}$, μ_{s}, G_{S}, μ_{L}, and Δ are nondimensional parameter of $O(1)$, and ϵ is a nondimensional wave amplitude which is sufficiently small compared with unity $(0<\epsilon\ll 1)$.

All the dependent variables are nondimensionalized and expanded in power series of ϵ, for example, the bubble radius R^* is expanded as

$$\frac{R^*}{R_0^*}=1+\epsilon R_1+\epsilon^2R_2+\cdots. \qquad (12)$$

5. Result

Equating the coefficients of like powers of ϵ in the resultant equations, we have the following set of linearized equations as the first-order equations:

$$\frac{\partial\alpha_1}{\partial t_0}-3\frac{\partial R_1}{\partial t_0}+\frac{\partial u_{\mathrm{G}1}}{\partial x_0}=0, \qquad (13)$$

$$\alpha_0\frac{\partial\alpha_1}{\partial t_0}-(1-\alpha_0)\frac{\partial u_{\mathrm{L}1}}{\partial x_0}=0, \qquad (14)$$

$$\beta_1\frac{\partial u_{\mathrm{G}1}}{\partial t_0}-\beta_1\frac{\partial u_{\mathrm{L}1}}{\partial t_0}-3\kappa p_{\mathrm{G}0}\frac{\partial R_1}{\partial x_0}=0, \qquad (15)$$

$$(1-\alpha_0+\beta_1\alpha_0)\frac{\partial u_{\mathrm{L}1}}{\partial t_0}-\beta_1\alpha_0\frac{\partial u_{\mathrm{G}1}}{\partial t_0} + (1-\alpha_0)\frac{\partial p_{\mathrm{L}1}}{\partial x_0}=0, \qquad (16)$$

$$3\kappa p_{G0}R_1+p_{L1}=0. \qquad (17)$$

Eliminating α_1, $u_{\mathrm{G}1}$, $u_{\mathrm{L}1}$, and $p_{\mathrm{L}1}$ from Eqs. (13)-(17), we can derive the linear wave equation for first-order perturbation of bubble radius, R_1:

$$\frac{\partial^2R_1}{\partial t_0^2}-\kappa p_{G0}\left(\frac{1}{1-\alpha_0}+\frac{1}{\beta_1}+\frac{1}{\alpha_0}\right)\frac{\partial^2R_1}{\partial x_0^2}=0. \qquad (18)$$

Hence, the effect of viscosity and rigidity of the shell do not affect and second-order of approximation is required.

6. Summary

The propagation of pressure waves in a liquid containing many microbubbles encapsulated by the viscoelastic shell is theoretically investigated. The linear wave equation from first-order approximation is derived and the effect of the shell does not appear. From second-order approximation as nonlinear propagation, shell viscosity and shell rigidity affect wave propagation.

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