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# Recent techniques on sound field simulation

# 最近の音場シミュレーション技法

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## 1. Introduction

Finite-difference time-domain (FDTD) method [1] is a most popular numerical method for non-stationary sound field analysis, since it has a simple algorithm that is easy to program. This article is a tutorial for analyzing sound wave propagation by the FDTD method. In particular, the CE-FDTD method [2-5] which is a higher accuracy version of FDTD method is mainly described.

## 2. Theory

2.1 Governing equations

For analysis of linear sound wave propagation without absorption, the following continuity equation and equation of motion are used.

$$\frac{\partial p}{\partial t} + \rho c_0^2 \nabla \cdot \boldsymbol{u} = 0 \quad (1)$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{\rho} \nabla p = 0 \quad (2)$$

where $p$ is sound pressure, $\boldsymbol{u} = (u, v)$ is particle velocity vector, $\rho$ is density, and $c_0$ is sound speed. On the other hand, the wave equation can be also used as follows.

$$\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = \nabla^2 p \quad (3)$$

So, which governing equation should be used for the sound field analysis? It depends on the problem to be solved. For most sound field analysis, the sound pressure is only required, so the wave equation (3) is sufficient. Eqs. (1) and (2) are used only when it is necessary to keep the particle velocity in the whole region as in the analysis of sound intensity distribution.

2.2 Discretization by FDTD method

First, we consider the discretization of eqs. (1) and (2) by the FDTD method in two-dimension. Since these equations are expressed by the first derivative in both time and space, the difference interval becomes $2\Delta$ in space or $2\Delta t$ in time using the central difference on the collocated grid as shown in Fig. 1 (a). In this case, the numerical accuracy is degraded, so the staggered grid as shown in Fig. (b) is generally used. Discretizing eqs. (1) and (2) on the staggered grid gives the following equations.

$$p_{i,j}^{n+1} = p_{i,j}^{n} - \chi\left(\bar{u}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - \bar{u}_{i-\frac{1}{2},j}^{n+\frac{1}{2}} + \bar{v}_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - \bar{v}_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}\right) \quad (4)$$

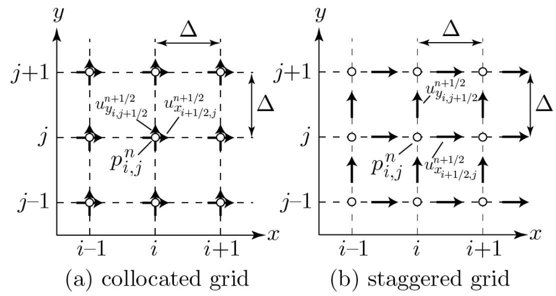



Fig.1 Two-dimensional FDTD grids.

$$\bar{u}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} = \bar{u}_{i+\frac{1}{2},j}^{n-\frac{1}{2}} - \chi\left(p_{i+1,j}^{n} - p_{i,j}^{n}\right) \quad (5)$$

$$\bar{v}_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} = \bar{v}_{i,j+\frac{1}{2}}^{n-\frac{1}{2}} - \chi\left(p_{i,j+1}^{n} - p_{i,j}^{n}\right) \quad (6)$$

where $p_{i,j}^{n}$ represents sound pressure on the grid $(x, y) = (i\Delta, j\Delta)$ at time $t = n\Delta t$, $(\bar{u}, \bar{v}) = \rho c_0 (u, v)$, and $\chi = c_0 \Delta t / \Delta$ is Courant number. The difference intervals can be kept at $\Delta$ and $\Delta t$ respectively by using the staggered grid, and the decrease in accuracy can be avoided. This discretization will be called the vector-type FDTD (vFDTD) method.

For the wave equation (3) another discretization can be given on the collocated grid as

$$p_{i,j}^{n+1} = 2p_{i,j}^{n} - p_{i,j}^{n-1} + \chi^2\left(\delta_x^2 + \delta_y^2\right)p_{i,j}^{n} \quad (7)$$

where

$$\delta_x^2 = p_{i+1,j}^{n} - 2p_{i,j}^{n} + p_{i-1,j}^{n},\ \delta_y^2 = p_{i,j+1}^{n} - 2p_{i,j}^{n} + p_{i,j-1}^{n} \quad (8)$$

This discretization will be called the scalar-type FDTD method (sFDTD). So which discretization is better for sound field analysis? We first consider the numerical accuracy. Eq. (7) can be obtained eliminating the particle velocity from eq. (4) using eqs. (5) and (6). So, it is found that both discretizations have the same accuracy. In vFDTD method, the particle velocity acts only as an intermediate variable in the program.

Next, we consider the memory usage required for analysis as shown in Table 1. vFDTD requires memory of $N$ for sound pressure and number of dimensions $\times N$ for particle velocity, where $N$ is number of grid points, while sFDTD requires $2N$ for the sound pressure $p^n$ and $p^{n-1}$ regardless of dimension ($p^{n+1}$ should be overwritten on $p^{n-1}$). The memory amount is proportional to the calculation time since most of the calculation time

Table 1 Memory usage required for analysis ($N$ is number of grid points).

| dimension | vFDTD | sFDTD |
|---|---|---|
| 1 | $2N$ | $2N$ |
| 2 | $3N$ | $2N$ |
| 3 | $4N$ | $2N$ |

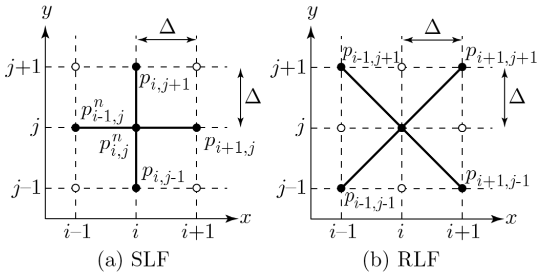


Fig.2 Grid point reference for the second-order difference.

depends on data transfer. So, it is found that sFDTD has advantage for sound field analysis.

2.3 Compact explicit (CE)-FDTD method [2-5]

In the standard FDTD method (SLF), the second-order difference is evaluated by the points in the axial direction as shown in Fig.2 (a). It is also possible to evaluate in the diagonal direction as shown in Fig.2 (b), which is called rotated leapfrog (RFL). The discretization form is given by

$$p_{i,j}^{n+1} = 2p_{i,j}^{n} - p_{i,j}^{n-1} + \chi^2 \delta_x^2 \delta_y^2 p_{i,j}^{n} \quad (9)$$

The compact explicit-FDTD method is derived by combining SLF and RFL as

$$p_{i,j}^{n+1} = 2p_{i,j}^{n} - p_{i,j}^{n-1} + \chi^2 \left(\delta_x^2 + \delta_y^2 + a\delta_x^2 \delta_y^2\right) p_{i,j}^{n} \quad (10)$$

where $a$ is a parameter to control accuracy. The greatest advantage of CE-FDTD is that the accuracy can be improved by increasing the number of evaluation points without memory increasing.

The three-dimensional form is given by

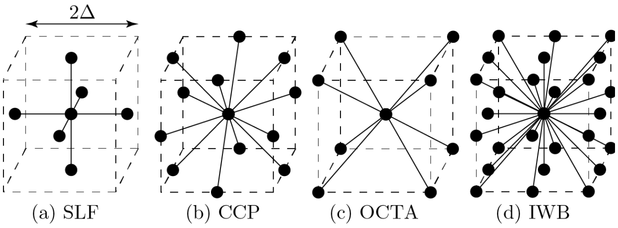
$$p_{i,j,k}^{n+1} = 2p_{i,j,k}^{n} - p_{i,j}^{n-1} + \chi^2 (\delta_x^2 + \delta_y^2 + \delta_z^2 + a(\delta_x^2 \delta_y^2 + \delta_y^2 \delta_z^2 + \delta_z^2 \delta_z^2) + b\delta_x^2 \delta_y^2 \delta_z^2) p_{i,j,k}^{n} \quad (11)$$

where $a$ and $b$ are independent parameters that control the accuracy, and various derivative methods are proposed as shown in Table 2, where $\chi_m$ is the upper limit of CFL, and $f_c$ is the cutoff frequency normalized by the sampling frequency. In the case of $a$=1/4, $b$=16, the scheme is called the interpolated wide band (IWB) which has an ideal characteristic that the cutoff frequency agrees with the Nyquist frequency.

Fig.3 shows the memory usage and the calculation time with GPU calculation for various derivative scheme when achieving the same bandwidth. In the figure, the values are normalized

Table 2 Numerical parameters for derivative schemes in the 3-D CE-FDTD method.

| scheme | $a$ | $b$ | $\chi_m$ | $f_c$ |
|---|---|---|---|---|
| SLF | 0 | 0 | $1/\sqrt{3}$ | 0.196 |
| CCP | 1/4 | 0 | 1 | 0.333 |
| OCTA | 1/2 | 1/4 | 1 | 0.25 |
| IWB | 1/4 | 1/16 | 1 | 0.5 |



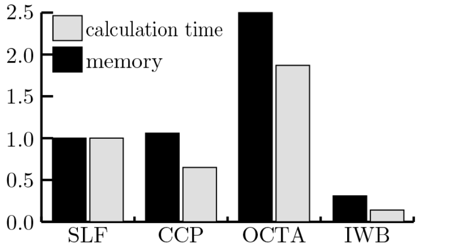



Fig.3 Computational performance with GPU calculation.

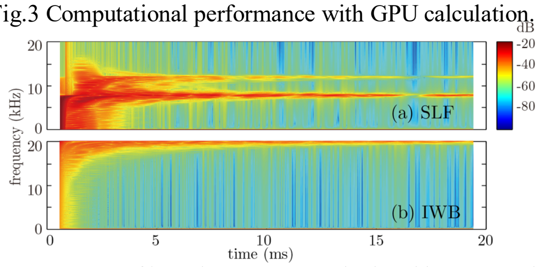


Fig.4 Spectrogram of impulse response calculated by SLF and IWB.

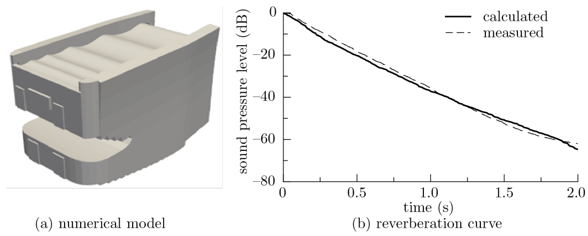


Fig.5 Yamaha Hall model and its reverberation curve calculated.

by SLF. It is found that the memory usage of IWB is smallest and is about 30% of that of SLF. It is also confirmed that the calculation time of IWB is shortest and is about 14% of that of SLF. This is because that the cutoff frequency of IWB is higher than that of SLF, so the sampling frequency of IWB can be set lower if the bandwidth is the same.

## 3. Numerical demonstrations

Fig. 4 shows the spectrograms of impulse response calculated by SLF and IWB schemes. The ringing caused by the numerical dispersion is observed in each scheme. Some obvious peaks are also observed in the spectrogram, corresponding to the cutoff frequency. It is experimentally found that the IWB scheme has the widest bandwidth. Fig.5 shows the reverberation curve of the Yamaha Hall. The reverberation curve agrees well with the measured one.

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