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# Experimental investigation of the dispersion relation of the Lamb wave propagating in a plate immersed one side surface in water on lower frequency region

片面に水が接する平板を伝搬するラム波の低周波領域における分散関係の実験的検討

Bunyu Tamura[1‡], Masashi Ishikawa[1], and Hideo Nishino[1]
([1]Tokushima Univ.)
田村文勇 [1‡]，石川真志 [1]，西野秀郎 [1] ([1] 徳大院)

## 1. Introduction

It is well known that the dispersion relation of the Lamb wave propagating in a plate with water on one side surface has a more complex structure than that in a free plate has complex one. Especially in the lower frequency region, it is confirmed theoretically that the lowest asymmetric mode of a free plate, the $A_0$, becomes the bifurcated $A_0$' mode and the quasi-Scholte (QS) mode in the water-loaded plate; however, the bifurcated $A_0$' mode emerges as theoretical solutions but, in many cases, takes no physical meanings based on the principle of causality. The theoretical investigations [1-3] of the Lamb wave in water-loaded plate have been carried out well, but the experimental verifications [3, 4] were not so many done well. Especially in the bifuracated mode in a metal plate, we could not find the experimental verifications in elsewhere. In this paper, experimental observations of the modes propagating in the SUS304 plates for different thicknesses were carried out using the air-coupled wedge sensors. It was shown in this paper that the propagation phenomena coincided well with the theoretical outcomes.

## 2. Characteristic equation and dispersion relation of the Lamb wave in water-loaded plate

Considering $x$ and $z$ axes as spatial coordinates in the propagation direction and the thickness respectively, the characteristic equation of the Lamb wave in the water-loaded plate on one side is as follows:

$$\begin{pmatrix} (k^2-\beta^2)e^{i\alpha d} & (k^2-\beta^2)e^{-i\alpha d} & 2\beta k e^{i\beta d} & -2\beta k e^{-i\beta d} & 0 \\ -2\alpha k e^{i\alpha d} & 2\alpha k e^{-i\alpha d} & (k^2-\beta^2)e^{i\beta d} & (k^2-\beta^2)e^{-i\beta d} & 0 \\ (k^2-\beta^2) & (k^2-\beta^2) & 2\beta k & -2\beta k & \frac{\rho_w}{\rho}k_t^2 \\ -2\alpha k & 2\alpha k & (k^2-\beta^2) & (k^2-\beta^2) & 0 \\ \alpha & -\alpha & -k & -k & \gamma \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \\ E \end{pmatrix} = 0 \quad (1)$$

where $A$, $B$, $C$, $D$, and $E$ are determined due to the eigenvibrations of the wave propagation. $\rho$, $\rho_w$, $k$, and $k_t$ are the densities of the plate material and

Corresponding: hidero.nishino@tokushima-u.ac.jp

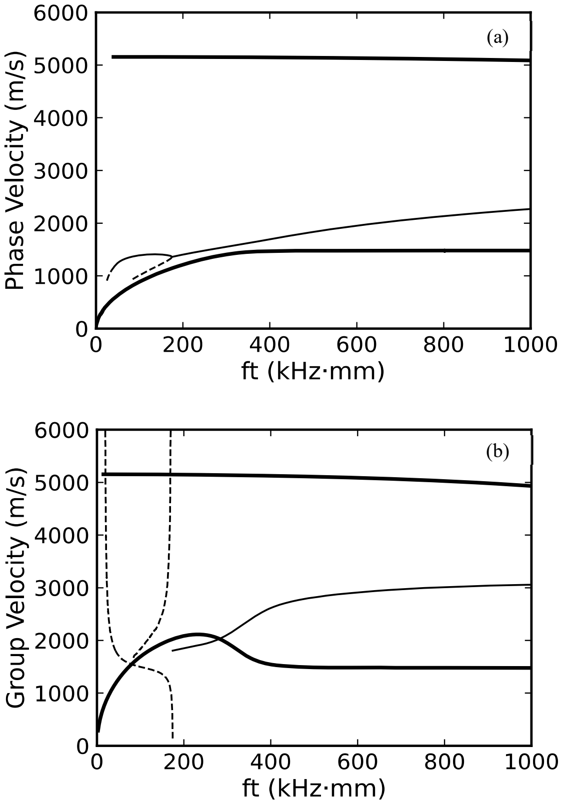


Fig. 1. Calculated dispersion curves for a SUS304 plate with thickness *t*. (a) phase velocity, (b) group velocity.

the water, the wavenumbers of the transverse and the Lamb wave, respectively. The $\alpha$, $\beta$, and $\gamma$ are projection components along with $z$ axis of longitudinal, transverse wave numbers of the Lamb wave, and wave number of the water, respectively. $2d$ is the plate thickness.

**Figures 1(a) and 1(b)** show the phase and group velocity dispersions of the SUS304 plate ($c_t$=3054 m/s, $c_l$=5692 m/s, u=7930 kg/m$^3$) immersed water ($c_w$=1480 m/s, $u_w$=1000 kg/m$^3$) in one side, respectively. The calculations were carried out with the Muller's method. The thick lines indicate the QS mode, and the black solid line indicated the $S_0$' and the $A_0$' modes for the water-loaded plate, respectively. It can be confirmed that the phase velocity of the $A_0$' mode is bifurcated and the group velocity of it takes interesting structures.

## 3. Experimental setup

The critical angle method with a pair of the air-coupled transducers ($f_c$=330 kHz) was used to generate and detect the Lamb wave. To measure the group velocities for f·t value ranging from 64 to 300 kHz·mm, the thickness of the plate was changed from 0.2 mm to 0.9 mm while the frequency was unchanged constantly at 330 kHz. The several pairs of the propagation distances and their times were obtained experimentally for accurate determinations of the group velocities of the Lamb waves.

## 4. Results

**Figure 3** shows, respectively, the examples of the time domain signals observed in the free plate (a), and the water-loaded plate (b), whose thickness is 0.4 mm. The $A_0$ and the QS mode waves were observed clearly in the signals; however, the bifurcated branch of the $A_0$' mode is not observed with the transducer pair. As shown before, this is due to the unphysical theoretical solution. The theoretical and experimental group velocity dispersions of the Lamb wave in the free plate are shown in **Fig. 4**. The experimental results agreed very well with the theory. **Figure 5** shows the theoretical and the experimental group velocities of the water-immersion modes. The lines and the markers indicate the theoretical and experimental results, respectively. All the experimental results for the group velocities coincided well with the theoretical outcomes.

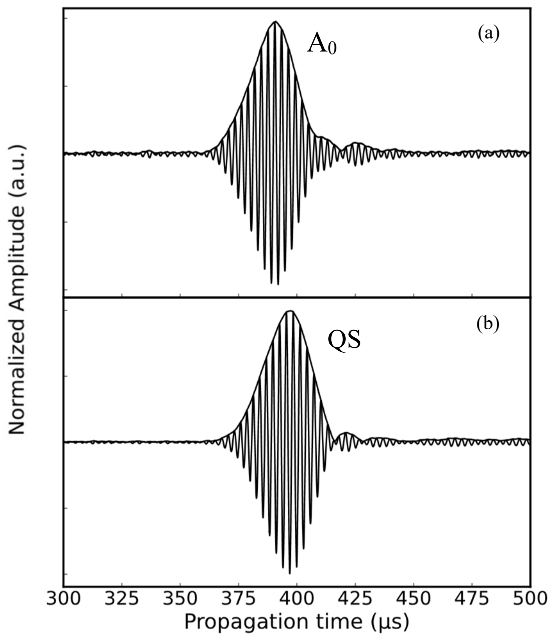


Fig. 3. Time domain waveform at 330 kHz in 0.4 mm thick plate. (a) free plate, (b) water-loaded plate.

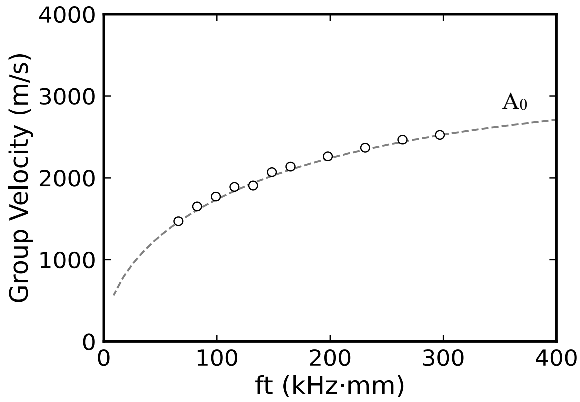


Fig. 4. Experimental (symbols) and calculated (dashed line) group velocity for free plate.

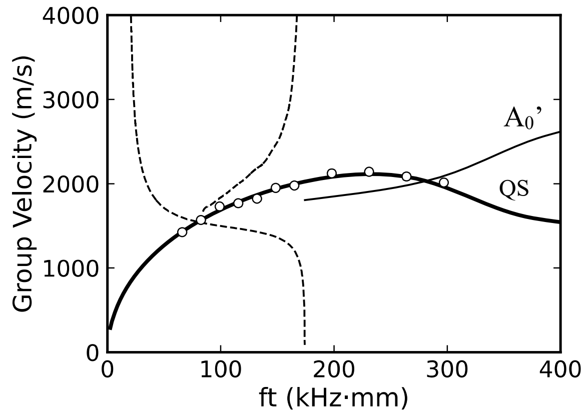


Fig. 5. Experimental (symbols) and calculated (full line) group velocity for SUS 304 plate loaded at one side with water.

## 5. Conclusions

In this paper, first, the theoretical characteristics of the Lamb wave propagating in the SUS304 plate immersed one side surface in water were briefly presented. It was confirmed that the bifurcated mode was observed in the lower frequency subsonic (against the water velocity) region. Also confirmed that the group velocities took curious structures. The experimental investigations to generate and detect the Lamb wave both in the water-loaded and in the free SUS304 plates were carried out using the critical angle method with the air-coupled transducer pair. The measured group velocities of the $A_0$ and the QS mode waves coincided well with the theoretical outcomes.

## Reference


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