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Regularization for Medical Ultrasound
医用超音波における正則化

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1. Introduction

Regularization is efficient processing to stabilize a large system of equations derived in various fields.[1] A well-known regularization is based on the Tikhonov's regularization[2]. Although the original regularization uses an L_2-norm of an unknown vector, an improved one uses an L_2-norm of a gradient and/or Laplacian of the unknown vector as well.[3] Recently, L_1-norm is also used. The singular-value decomposition (SVD) also outputs a useful result, although an amount of calculations is larger than the regularizations. Downsampling is also well performed. Commonly, amplification of high frequency noises is suppressed.

In our case, various new regularizations are performed for superresolution of a medical ultrasound (US) echo image (eg., ref. 4), US Doppler measurement such as blood flow and/or tissue motion/strain (eg., old ref. 5), thermal and mechanical reconstructions (eg., old refs. 6 and 7), etc. We are also performing comparison of the above-mentioned regularization methods, the maximum a posteriori (MAP) method,[8,9] the maximum likelihood (ML) method[8,9] and the weighted least squares method[1]. In this report, the MAP is also regraded as a kind of regularization.

2. New regularizations

2.1 Approaches

The traditional regularizations are performed such that the errors of equations derived for targets and those of direct relations to targets (penalty terms) are same.[2,3] In contrast, our developed regularizations are performed such that the penalty terms are weighted properly using regularization parameters based on the statistically evaluated errors of penalty terms.[4-7] For all the stabilizations including the traditional ones, we had recognized that it is computationally efficient to perform weighting multiplications onto the equations and/or the penalty terms instead of traditional weighting divisions.

Similarly to the traditional regularizations, our developed regularizations can also be performed empirically. However, the regularization parameters can also be systematically determined using the generalized cross validation (GCV) method, etc. by mathematically considering the condition number of a matrix to be inverted. Alternatively, we can also perform the determination automatically by controlling the regularization parameters with evaluating a signal-to-noise ratio (SNR) and/or a contrast-to-noise ratio (CNR) as a metric [eg., 5].

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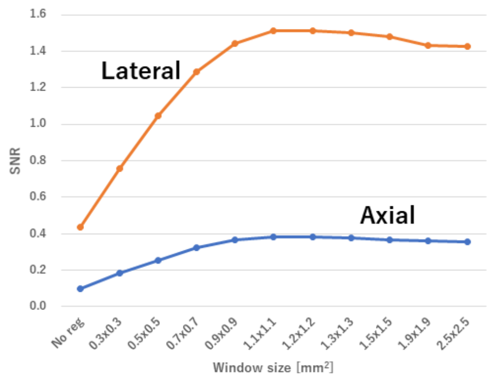

Fig. 1 SNRs of axial and lateral strains versus window size used for estimating variances.

2.2 Doppler measurements and reconstructions

For blood flow and/or tissue motion/strain Doppler measurement, and thermal and mechanical reconstructions, we have been performing all the regularizations. For the real-world displacement measurement, all the regularizations are more effective than the ML and weighted least squares method. Particularly, our developed regularization and the MAP are effective. In our displacement vector measurement, we apply the regularizations to the respective displacement components, which we refer to as the displacement-component-dependent regularizations. Our developed regularization is compared with the MAP for uses of the L2-norms of a target vector[9] and the Laplacian (a modified MAP)[10]. The stabilization and calculations of MAPs are much more intense than those of our developed regularizations, which can result in extra-regularization. Now a proper window size used for the locally stationary variance estimation of penalty terms is also examined. **Fig. 1** shows for an agar phantom laterally compressed the SNRs of axial and lateral strains measured with changing the window size using our previously developed multidimensional autocorrelation method (MAM) with a moving-average size, 0.5 × 0.5 mm^2 (7.5 MHz). For such a displacement vector measurement, our previously developed multidimensional cross-spectral phase gradient method (MCSPGM) is also used (omitted here). Now, the performance of a gradient penalty term is also compared for the MAM and MCSPGM.

As another approach for estimating the variance is to use the Cramer-Rao Lower Bound (CRLB, i.e., SD: standard deviation)[7]:

$$\sigma_{\mathrm{CRLB}} = \frac{3}{2\pi^2 \mathrm{T}(\mathrm{B}^3 + 12\mathrm{Bf}^2)} \left\{ \left(1 + \frac{1}{\mathrm{SNR_c}}\right)^2 - 1 \right\}, \quad (1)$$

where T is a local echo data length or a moving-average window size; f is a frequency; B is a bandwidth; SNRc is combined echo SNR and correlation SNR[7]; or a power Doppler (SD)[8,11]:

$$\sigma_{\text{power Dopp}} = \sqrt{2\left[1 - \frac{|R(\Delta t)|}{R(0)}\right]}, \quad (2)$$

where R(Δt) is an autocorrelation function with a pulse repetition interval Δt. These approaches perform a priori statistic evaluation instead of a posteriori one. When performing the displacement vector measurement, we actually perform a lateral modulation by performing plural steered beamformings or dividing spectra to yield plural quasi-steered beams. Thus, the variance estimations can be performed for respective orthogonal directions, i.e., axial, lateral and elevational directions. Thus, similarly to the above a posteriori regularizations, the displacement-component-dependent regularization can be performed.

Since the stationary estimation, the CRLB and the power Doppler can also be used for estimating a displacement variance in each steered beam direction, the variance estimate can also be divided into the respective orthogonal components simultaneously. Thus, the estimate can also be used for weighting a penalty term corresponding to each beam solo or with a combination of the above or simultaneous component estimates.

All the stabilization approaches can also be applied to the thermal and mechanical reconstructions. Specially, the variance of a displacement or a strain tensor component can also be used for the stabilization (e.g., ref. 7).

2.3 Superresolution

The regularizations, MAPs, ML and weighted least squares estimation can also be performed for the superresolution, i.e., when performing inverse filtering in a frequency (fast calculation) or spatial (slow one) domain. Our point spread function (PSF)

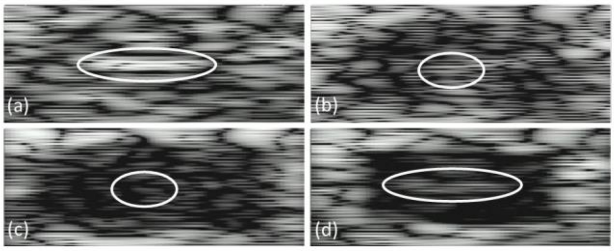

Fig. 2 For an agar phantom, (a) raw image, and inverse filtered images: (b) regularized image, and MAP images (c) with proper regularization and (d) with extra regularization.

is estimated using an averaged or nonaveraged autocorrelation function.[1] The spatial inverse filtering achieves spatially shift variant processing by estimating the PSF, for instance, at each depth. The performances are compared each other for an agar phantom having an intense scatter (**Fig. 2**). For real-world rf-echo data, the regularizations and the MAPs yield favorable results. The achieved spatial resolutions can also be evaluated using an autocorrelation function (omitted).

3. Conclusions

Our developed regularizations and MAPs are effective in real-world applications for the displacement measurement, and thermal and mechanical reconstructions. Although the practical performances are evaluated at first, the effect of noise amount in measured data also need to be evaluated specifically, for instance, by using an artificial noise. In the near future, the performances will also be compared with deep learning (DL) displacement/strain estimation, and speckle reduction and superresolution.

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