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Weakly nonlinear theory on ultrasound propagation in liquids containing microbubbles coated by a lipid shell
脂質膜で覆われた球形気泡を多数含む液体中における
超音波の非線形理論解析

Mitsuhiro Honda[1†], Tetsuya Kanagawa[2], and Yusei Kikuchi[3] ([1]Eng. & Syst., Univ. of Tsukuba; [2] Inf. & Syst., Univ. of Tsukuba; [3] Grad. Sch. Syst. Inf. Eng., Univ. of Tsukuba)
本多満洋 [1†]，金川哲也 [2]，菊池勇成 [3] （[1]筑波大理工，[2]筑波大シス情，[3]筑波大院シス情）

1. Introduction

In recent years, applications of ultrasound in the medical field have received significant attention and have been developed. The use of microbubbles as ultrasound contrast agents (UCAs) [1] has drastically improved the resolution of images.

In most UCAs, the microbubble is stabilized against dissolution by being covered by a lipid shell, membrane, and so on; i.e., encapsulated microbubble (UCA). Church [2] and Hoff et al. [3] established the theory of oscillations of an UCA and derived the equation of motion for radial oscillations. However, these studies were restricted to single UCA. Therefore, the theory for many UCAs in the ultrasound field should be established since many UCAs are usually used for diagnosis in the clinical field. Although our previous study [4] clarified propagation properties of nonlinear ultrasound in liquids containing many UCAs, it is restricted under a low-frequency long wave band. Focusing on a resonance band, the ultrasound may help to resolute body parts more precisely and nonlinear component of that which is enhanced induce higher harmonics which improves the resolution of images.

The present target is high-frequency short wave band, as an extension of previous low-frequency long wave [4]. We then clarify the effect of the shell rigidity, surface tension and shell viscosity to various propagation properties of nonlinear ultrasound in liquids containing many UCAs and compare the result to previous result [4].

2. Problem statement

Weakly nonlinear (i.e., finite but small amplitude [4]) propagation of plane progressive ultrasound in an initially quiescent liquid uniformly containing many spherically UCAs (encapsulated microbubbles) is theoretically studied.

At an initial state, all the dependent variables including the bubble radius and the void fraction are assumed to be uniform. The following assumptions are used: (i) UCAs do not coalesce, break up, appear, and disappear, (ii) UCA oscillations are spherically symmetric; (iii) liquid compressibility, viscosity of gas inside the UCAs, and the thermal conductivities of both phases are neglected.

To describe the motion of many UCAs is the present novelty. The surrounding shell of each UCA is assumed as a visco-elastic (Kelvin-Voigt model [4]) body. We then the Church [2] (or Hoff et al. [3]) model as equation of motion for oscillations for single UCA:

$$
\begin{aligned}
&\rho_{\mathrm{L}}^{*}\left[R^{*}\frac{\mathrm{D}_{\mathrm{G}}^{2}R^{*}}{\mathrm{D}t^{*2}}+\frac{3}{2}\left(\frac{\mathrm{D}_{\mathrm{G}}R^{*}}{\mathrm{D}t^{*}}\right)^{2}\right] \\
&= p_{\mathrm{G0}}^{*}\left[\left(\frac{R_{0}^{*}}{R^{*}}\right)^{3\gamma}-1\right]-P_{L}^{*}-\frac{2\sigma^{*}}{R^{*}}\left(1-\frac{R_{0}^{*4}}{R^{*4}}\right) \\
&\qquad -\frac{4\mu_{\mathrm{L}}^{*}}{R^{*}}\frac{\mathrm{D}_{\mathrm{G}}R^{*}}{\mathrm{D}t^{*}}-12\mu_{\mathrm{S}}^{*}\frac{d_{\mathrm{S0}}^{*}R_{0}^{*2}}{R^{*4}}\frac{\mathrm{D}_{\mathrm{G}}R^{*}}{\mathrm{D}t^{*}} \\
&\qquad -12G_{\mathrm{S}}^{*}\frac{d_{\mathrm{S0}}^{*}R_{0}^{*2}}{R^{*3}}\left(1-\frac{R_{0}^{*}}{R^{*}}\right), \qquad (1)
\end{aligned}
$$

where R^* is the bubble radius, G_{S}^* shell rigidity, μ_{S}^* shell viscosity, p_{G0}^* initial gas pressure, p_L^* driving pressure of liquid, μ_{L}^* liquid viscosity, d_{S0}^* initial shell thickness, γ polytropic exponent, and σ^* initial gas pressure. Noting that Rayleigh-Plesset type equation is usually utilized for uncoated bubble.

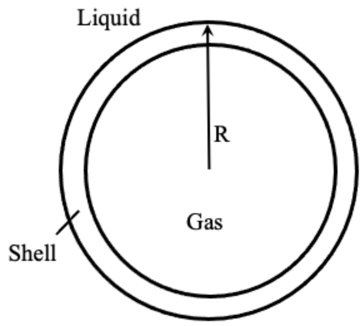

Fig. 1 Conceptual illustration of an UCA (encapsulated bubble).

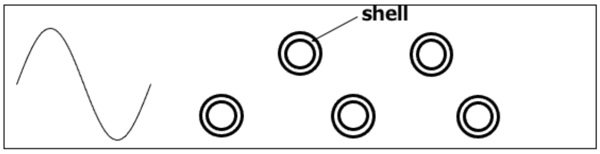

Fig. 2 Conceptual illustration of ultrasound radiation in liquid containing many UCAs.

Although our previous target is a low-frequency long wave [4], our present target is a high-frequency short wave. To focus on this range, a set of nondimensional ratios among the physical parameters is determined:

$$\frac{R_0^*}{L^*} = \Delta, \qquad \frac{\omega^*}{\omega_B^*} = \Omega, \tag{2}$$

where R_0^* is initial bubble radius, L^* is the typical wavelength of the wave, ω^* is an angular frequency of the sound source, ω_B^* is the natural angular frequency, and Δ and Ω are constants of $O(1)$.

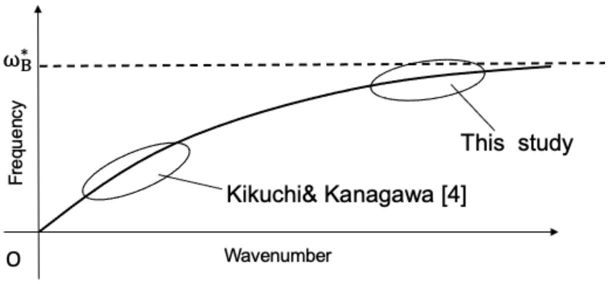

Fig. 3 The dispersion relation in bubbly liquids [4,5].

3. Theoretical analysis

The conservation equations of mass and momentum for gas and liquid phases for bubbly medium are introduced:

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) = 0, \tag{3}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] = 0, \tag{4}$$

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}\left(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^{*2}\right) + \alpha\frac{\partial p_{\mathrm{G}}^*}{\partial x^*} = F^*, \tag{5}$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^{*2}\right] + (1-\alpha)\frac{\partial p_{\mathrm{L}}^*}{\partial x^*} + P^*\frac{\partial \alpha}{\partial x^*} = -F^* \tag{6}$$

where t^* is the time, x^* space coordinate normal to the wave front, α void fraction $(0 < \alpha < 1)$, ρ^* density, u^* fluid velocity, p^* pressure, and F^* interfacial momentum transport; the subscripts G and L denote volume-averaged variables in the gas and liquid phases, respectively. In addition to the volume-averaged pressure p_{G}^* and p_{L}^*, the liquid pressure averaged on the bubble-liquid interface, P^*, is introduced.

As in Eq. (2), the other nondimensionalized rations are estimated to describe high-frequency short wave band in Fig. 3 as fllows:

$$\frac{d_{\mathrm{S0}}^*}{R_0^*} = d_{\mathrm{S0}}\,\epsilon^{\frac{3}{2}}, \qquad \frac{\mu_{\mathrm{S}}^*}{\rho_{\mathrm{L0}}^* U^{*2} T^*} = \mu_{\mathrm{S}}\sqrt{\epsilon},$$

$$\frac{G_{\mathrm{S}}^*}{\rho_{\mathrm{L0}}^* U^{*2}} = G_{\mathrm{S}}\sqrt{\epsilon}, \qquad \frac{\mu_L^*}{\rho_{\mathrm{L0}}^* U^{*2} T^*} = \mu_{\mathrm{L}}\epsilon^2, \tag{7}$$

where d_{S0}, μ_{S}, G_{S}, μ_{L} are constants of $O(1)$, ϵ is a nondimensional wave amplitude sufficiently small compared with unity $(0 < \epsilon \ll 1)$.

Substituting Eqs. (2) and (7) into Eqs. (1), (3)-(6) and supplementary equations in Ref. [4] and equating the coefficients of like powers of ϵ, ϵ^2, ϵ^3 in the resultant equations, we obtain three approximation equations; the detail of calculation is not shown here for the economy of space. As a result, we have the nonlinear Schrödinger equation,

$$i\frac{\partial A}{\partial \tau} + \frac{q}{2}\frac{\partial^2 \mathrm{A}}{\partial \xi^2} + \nu_1|\mathrm{A}|^2 A + i\nu_2 A + \nu_3 A = 0, \tag{8}$$

$$\tau \equiv \epsilon^2 t, \qquad \xi \equiv \epsilon\left(x - v_g t\right), \tag{9}$$

where A is the complex amplitude, $q/2$ denotes the dispersion coefficient, ν_1 is the nonlinear coefficient, ν_2 is the dissipation coefficient, and ν_3 is advection coefficient, and v_g is the group velocity. The explicit forms of ν_1, ν_2, and ν_3 are not shown for the economy of space. The first, second, third, and fourth terms in Eq. (9) represent dispersion, nonlinear, dissipation, and advection effects, respectively.

4. Summary

To clarify the ultrasound propagation in liquids containing many UCAs, we extend our previous target as low-frequency long wave to high-frequency short wave as a resonance band. The shell rigidity, surface tension, and shell viscosity contributed to the advection, nonlinear, and dissipation effects, respectively, as in Ref. [4]. Then, propagation feature of ultrasound with a short wave is similar with that with a long wave. The detailed result will appear in presentation and a presentation and a forthcoming paper.

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