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Real and complex asymmetric parameters of Fano resonance in a simple classical harmonic oscillator system

古典的調和振動子系に生じるファノ共鳴の
実数および複素数の非対称パラメータ

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1. Introduction

Recently, much attention has been devoted to a class of sonic or phononic crystals with locally resonant structural units. These crystals exhibit frequency gaps for lattice constants considerably smaller than the relevant wavelength. As simple examples of such artificial crystals, one- dimensional phononic crystals consisting of solid and fluid layers have been studied because of their setting and tuning advantages. One of the characteristics of the frequency gap caused by the local resonance is that a frequency giving a transmittance of 0 already exists even in the case of a single period structure. This is quite different from the Bragg gap, where the transmittance decreases as the number of periods increases.

In our previous works [1, 2], we theoretically studied the resonant transmission of acoustic waves propagating through a solid slab in a fluid. When acoustic waves are incident on the interface perpendicularly, Bright–Wigner type resonances occur. When acoustic waves are incident at an angle, on the other hand, Fano resonances [3,4] are observed. We demonstrated that an explicit expression for the resonant profile can be derived for this simple phononic system. In particular, an analytical expression for the Fano parameter was derived.

The Fano resonance is due to the constructive and destructive interference of a discrete localized state with a continuum of propagating modes that share the same frequency. In fact, Fano resonances have been reported in photonic crystals, plasmonic nanostructures, metal-slit superlattices, and phononic metamaterials.

Suppression of the systems response due to destructive interference in Fano resonance was explained using a simple model consisting of two weakly coupled harmonic oscillators [4, 5], but even in such a simple system, the expression for the resonance profile was not derived directly from the equation of motion. Also, Fano parameters have been implicitly treated as real numbers in many previous studies.

In the present work, we theoretically examine the resonance generated in two weakly coupled harmonic oscillators in detail. We analytically calculate the amplitude profiles near the resonant frequencies and show that the resonance can be generally described by a Fano formula with a complex Fano parameter.

2. Theoretical Model

In this study, we examine the dynamics of a pair of harmonic oscillators connected by a weak spring. This system is specified with parameters ω_i, γ_i, and $V(\neq 0)$, which are eigenfrequency of the oscillator i $(=1,2)$ when there is no friction, the friction coefficient, and coupling constant, respectively. We also assume that a periodic external force with a frequency ω is acts on the oscillator 1. The stationary solutions can be obtained in the form of $x_i = c_i e^{i\omega t}$. We calculated the complex amplitude c_i analytically and derived the Fano formula near the resonant frequency, where the Fano parameter is generally represented by a complex number.

3. Results and discussions

As a numerical example, we show in Fig. 1 $|c_1|$ and $|c_2|$ calculated for $\omega_1 = 1.0$, $\omega_2 = 2.0$, $\gamma_1 = 0.5$, $\gamma_2 = 0$, and $V = 0.1$. $|c_1|$ has a Fano-type asymmetric profile around a resonant

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frequency slightly away from ω_2. On the other hand, $|c_2|$ has a Lorentz-type profile around the same resonance frequency as $|c_1|$. The calculated resonant frequency, resonant width and Fano parameter is $\omega_{reso} = 2.014$, $\Gamma = 0.01$, and $q = 1.5$, respectively. The solid line in Fig. 1 is an exact solution, and the dotted line represents the profile obtained by using the Fano formula. For $\gamma_2 = 0$, the Fano parameter is sown to be real.

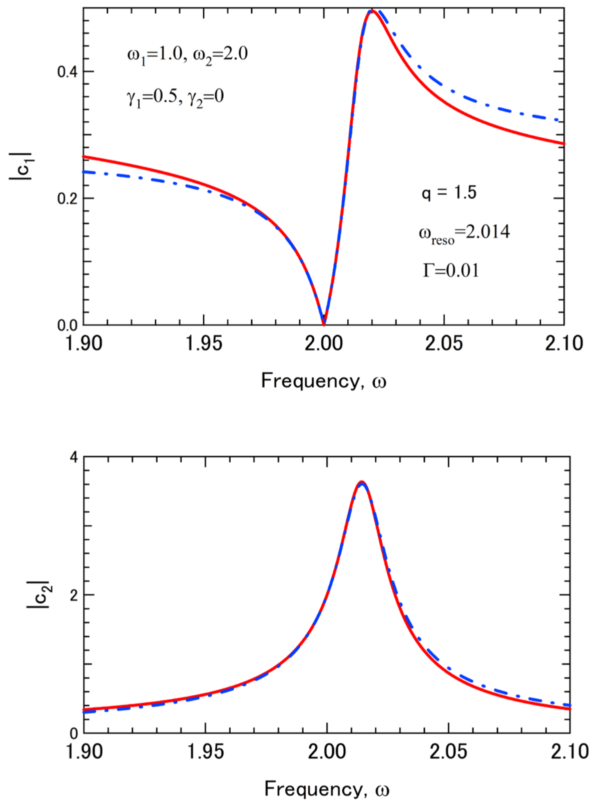

Fig. 1 Amplitudes of the first and second oscillators for $\omega_1 = 1.0$, $\omega_2 = 2.0$, $\gamma_1 = 0.5$, $\gamma_2 = 0$, and $V = 0.1$.

Fig. 2 is calculated for $\omega_1 = 1.0$, $\omega_2 = 2.0$, $\gamma_1 = 0.5$, $\gamma_2 = 0.01$, and $V = 0.1$. When $\gamma_2 \neq 0$, the Fano parameter is shown to be complex. The calculated resonant parameters are $\omega_{reso} = 2.014$, $\Gamma = 0.02$, and $q = 0.7 - 0.5i$.

The expression and its meaning of the complex Fano parameter will be discussed in detail in the presentation. Although the system considered in the present work is very simple, it is useful not only for understanding Fano resonance, but also for studying more complicated systems. This is because Fano resonance is one of the fundamental wave phenomena.

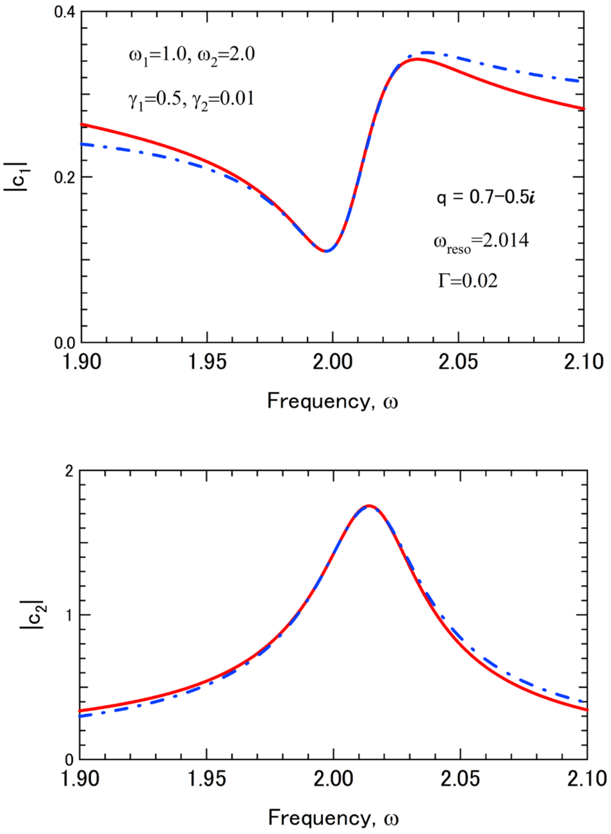

Fig. 2 Amplitudes of the first and second oscillators for $\omega_1 = 1.0$, $\omega_2 = 2.0$, $\gamma_1 = 0.5$, $\gamma_2 = 0.01$, $V = 0.1$.

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