

Finite element modeling of acoustic transmission and reflection loss in ultrasound transducer

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1. Introduction

High-frequency ultrasonic transducers have enormous applications in industrial to bio medical fields. They are used in medical imaging and tissue ablation as well as in industries for quality inspection, material characterization, detection of defects, flaws and voids in electronic components and composite structures¹⁻⁵. When the acoustic waves propagate between mediums, some parts of the wave get transmitted while some get reflected at the interface. The knowledge about the waves reflection and transmission losses is an essential parameter in microscopy domain. The theoretical equations for finding reflection and transmission coefficients are suitable for a smooth plane boundary but fail for the curved or the rough boundaries. Here we present FEM based modeling of these losses for a focused ultrasonic transducer having a spherical cavity at the water interface and present a theoretical validation of the model.

2. Theoretical analysis

Reflection of ultrasonic waves occurs at the boundary of two mediums because of the difference in the acoustic impedance (Z) between them. The impedance of a medium is defined as the product of its density (ρ) and the speed of sound (v) in that medium, i.e., $Z = \rho v$. This difference in impedance between two mediums is referred to as impedance mismatch. The greater this mismatch, the greater the amount of energy reflected from the boundary and the lesser is the energy that propagates in the subsequent medium. The boundary phenomenon for longitudinal ultrasound, i.e., reflection and transmission at the interface of two mediums, is similar to that of the light (Electromagnetic waves). From **Fig. 1**, the angle of reflection is equal to that of the angle of incidence, $\theta_i = \theta_r$ as well as the transmitted wave angle, θ_t , satisfies the condition of wavefront coherence at the boundary which yields Snell's law in acoustic, i.e.

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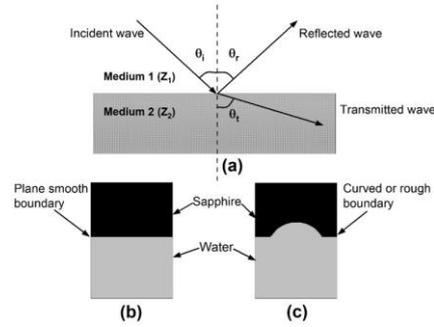


Fig. 1 (a) Ray diagram representing the reflection and transmission of wave at boundary. (b) Plane smooth lens-coupling medium interface. (c) Curved interface (focused lens).

$(\sin\theta_i/\sin\theta_t) = (v_1/v_2)$. The propagation of these waves through the boundary should not create any discontinuities in the particle's velocity or pressure. This condition leads to the following relationship for reflection (C_r) and transmission (C_t) coefficient^{6, 7}

$$C_r = \frac{P_r}{P_i} = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \quad (1)$$

$$C_t = \frac{P_t}{P_i} = \frac{2Z_2 \cos\theta_i}{Z_2 \cos\theta_i + Z_1 \cos\theta_t} \quad (2)$$

where P_i is the pressure of the incident wave, and P_r and P_t are the pressure amplitudes of reflected and transmitted waves, respectively. The reflection and transmission coefficients are dimensionless quantities and represent that the reflected wave has $(C_r \times 100)$ % of the sound pressure of the incident wave, and the transmitted wave has $(C_t \times 100)$ % pressure amplitudes. A better approach to describe these boundary phenomena can be in terms of energy rather than pressure amplitude. The intensity (i.e., the energy per unit area per unit time) of these waves is related to the pressure amplitude by the following equation^{6, 7}

$$I = \frac{P^2}{2Z} \quad (3)$$

3. Simulation and Results

Theoretically, the loss coefficients are calculated from Eq. (1) and (2). The above formulas are good for waves striking a flat (plane) and smooth boundary at normal or oblique incidence but fails for

curved or rough boundaries. In this study, we present a time-domain based FEM model for these loss calculations. Here we have performed loss calculations at the lens-water interface for a focused ultrasonic transducer. First, the lens surface in contact with water is considered planar, and the simulated result is compared with theoretical calculations, and then the same thing is performed for a focused lens that has a spherical cavity at the interface. The material values used are $\rho_1 = 3980 \text{ kg/m}^3$, $v_1 = 10,000 \text{ m/s}$, $\rho_2 = 998 \text{ kg/m}^3$, $v_2 = 1481.5 \text{ m/s}$, where the indices 1 and 2 are for the lens and water medium respectively. Numerically the coefficients are calculated using the pressure amplitudes of the incident, reflected and transmitted wave simulated in commercially available software COMSOL Multiphysics. For simplicity, waves are considered striking at normal incidence, i.e., $\theta_i = \theta_r = 0$. The result is shown in **Table 1**.

It can be seen that for a planar interface the difference between the theoretical and the simulated

Table 1. Comparison of theoretical and simulated losses for a smooth and curved boundary at lens-water interface

Lens-water interface (Pa)	Reflection coefficient (C_r)		Transmission coefficient (C_t)	
	Theo.	Sim.	Theo.	Sim.
Flat surface $P_i = 0.4875$ $P_r = 0.4464$ $P_t = 0.035$	0.9284	0.9158	0.0716	0.0718
Curved surface $P_i = 0.4875$ $P_r = 0.37$ $P_t = 0.03$	0.9284	0.7590	0.0716	0.0615

result is $\approx 1\%$. Thus, the correctness of the model is validated. But, the difference for the spherical curved interface $\approx 20\%$. This huge deviation in the results is because of the scattering (diffraction) of the wave from the curved part (spherical cavity) of the lens, which is not accounted for in the theoretical formula. Thus, the theoretical formula fails for the calculation of reflection and transmission loss for an ultrasonic transducer.

The COMSOL model here, is a 2D axisymmetric model computed in the time domain. The lens dimensions and material properties are taken from the literature⁸⁾. Acoustic solid interaction, transient physics is used, which adds the Multiphysics coupling for acoustic structure interaction. Plane wave radiation boundary condition is used to absorb the outgoing waves. The input excitation is a ricker pulse of frequency 250 MHz given by the following equation

$$V = (1 - 2\pi^2 f^2 (t - t_0)^2) e^{-(\pi^2 f^2 (t - t_0)^2)} \quad (4)$$

Where f is the excitation frequency, and t_0 is $1/f$. The

incident reflected, and the transmitted wave propagation can be seen in **Fig. 2** for the two mentioned surfaces.

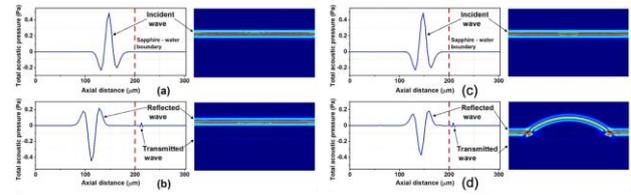


Fig. 2. Incident ricker pulse (a) at plane smooth boundary, (c) at curved boundary. Reflected and transmitted ricker pulse (b) at plane smooth boundary, (d) at curved boundary.

Since the wave travels from a higher impedance material to a lower impedance, there is a phase reversal of π in the reflected wave.

4. Conclusion

In conclusion, we have demonstrated the use of FEM model for acoustic loss calculations at the boundary of two mediums. Here we have used the model for the calculation of reflection and transmission loss for an ultrasonic transducer. In general, the model can be used for any arbitrarily shaped boundary for simulating the reflected and transmitted wave, which can be used to calculate useful parameters like these loss coefficients.

Acknowledgement

Authors acknowledge the funding from Research Council of Norway, INTPART project (309802).

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