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High-power characteristic evaluation for piezoelectric 31 effect and 33 effect using 5th order elastic constant

5 次弾性振動を考慮した圧電横効果および縦効果のハイパワー特性評価

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1. Introduction

Piezoelectric transducers are applied in various devices and they are driven under high power condition. Nonlinear effect such as “jumping phenomena” in current, hysteresis of admittance curve and temperature rise of piezoelectric transducer under high power driving are well-known[1]; however, they cannot be evaluated by a traditional piezoelectric equation because it is based on only a linear piezoelectric effect. To evaluate the actual performance of piezoelectric transducer under high power condition, the influence of the higher order elasticity must be clarified. Our research group has developed the evaluation method of the piezoelectric high power characteristics based on only 3rd order elasticity[2]. However, 5th order elastic vibration cannot be negligible under large stress.

In this research, the 3rd order elastic constant of 31 and 33 effect piezoelectric transducer was measured by admittance curve fitting under large stress condition over 30 MPa. The 3rd order elastic constant showed the stress dependency in both cases. Using the nonlinear transfer matrix method considering 5th order elastic constant, the value of 5th order elastic constant and the stress dependency of 3rd order elastic constant were evaluated.

2. Modelling of nonlinear piezoelectric vibration

2.1 Nonlinear piezoelectric equation

Piezoelectric equation of 33 effect with 3rd and 5th order elastic terms is expressed as equations (1)-(2):

$$T_3 = \overline{c_{33}^E} S_3 + c_{33(3)}^E S_3^3 + c_{33(5)}^E S_3^5 - \overline{e_{33}} E_3 \qquad (1)$$

$$D_3 = \overline{e_{33}} S_3 + \overline{\varepsilon_{33}^S} E_3 \qquad (2)$$

where T_3, S_3, E_3, D_3 are strain, stress, electric field and electric flux density; $\overline{c_{33}^E} = \frac{1}{s_{33}^E}$, $c_{33(3)}^E$, $c_{33(5)}^E$, $\overline{e_{33}} = \frac{d_{33}}{s_{33}^E}$, $\overline{\varepsilon_{33}^S} = \varepsilon_{33}^T - \frac{d_{33}^2}{s_{11}^E}$ are linear elastic constant, 3rd order elastic constant, 5th order elastic constant, piezoelectric constant and dielectric constant. 3rd order term $c_{33(3)}^E S_3^3$ and 5th order term $c_{33(5)}^E S_3^5$ are considered and other higher terms are not taken into account because the even-ordered terms don’t affect the signal with driving frequency and the influence of larger than 7th order terms is negligible. Elastic constants $\overline{c_{33}^E}$, $c_{33(3)}^E$ and $c_{33(5)}^E$ were treated as complex number as equation (3)-(5):

$$\overline{c_{33}^E} = \mathrm{Re}\left(\overline{c_{33}^E}\right) + j\mathrm{Im}\left(\overline{c_{33}^E}\right) \qquad (3)$$

$$c_{33(3)}^E = \mathrm{Re}\left(c_{33(3)}^E\right) + j\mathrm{Im}\left(c_{33(3)}^E\right) \qquad (4)$$

$$c_{33(5)}^E = \mathrm{Re}\left(c_{33(5)}^E\right) + j\mathrm{Im}\left(c_{33(5)}^E\right) \qquad (5)$$

2.2 Nonlinear transfer matrix

From the nonlinear piezoelectric equation (1)-(2), wave equation is expressed as equation (6):

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad (6)$$

where displacement u is expressed as :

$$u(x,t) = u_x e^{j\omega t} \qquad (7)$$

and sound velocity $c = \sqrt{\frac{c_{33}'}{\rho}}$. The parameter c_{33}' is nonlinear elastic constant which depends on internal stress. It is expressed as equation (8):

$$c_{33}'(x) = \overline{c_{33}^D} + \frac{3}{4} c_{33(3)}^E \left(\frac{\partial u_x}{\partial x}\right)^2 + \frac{5}{8} c_{33(5)}^E \left(\frac{\partial u_x}{\partial x}\right)^4 \qquad (8)$$

As shown in equation (5), nonlinear elastic constant $c_{33}'(x)$ depends on strain $\frac{\partial u_x}{\partial x}$. Therefore, transfer matrix is used to calculate strain distribution. Transfer matrix is expressed as equation (9):

$$\begin{pmatrix} F_n \\ v_n \\ D_3 \end{pmatrix} = \begin{pmatrix} -\frac{a}{b} & \frac{-a^2+b^2}{b} & \frac{A(a+b)}{b} \\ -\frac{1}{b} & -\frac{a}{b} & \frac{A}{b} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_{n+1} \\ v_{n+1} \\ D_3 \end{pmatrix} \qquad (9)$$

where $a = jSZ\left\{\tan\left(\frac{kl}{2}\right) - \frac{1}{\sin(kl)}\right\}$, $b = \frac{jSZ}{\sin(kl)}$, wave number $k = \frac{\omega}{c}$, force factor $A = \frac{wh\overline{e_{33}}}{\overline{\varepsilon_{33}^S}}$, acoustic impedance $Z = \rho c$, cross-sectional area $S = wh$ and w, h are width, height of the transducer. F_n, F_{n+1} are force and v_n, v_{n+1} are velocity at each surface. In the case of piezoelectric 33 effect transducer, electric field depends on internal strain because electric field and strain are parallel. On the other

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hand, electric flux density D_3 doesn't depend on position. The parameter D_3 is obtained as following equation

$$D_3 = \frac{\overline{\varepsilon_{33}^S}}{l} V + \frac{\overline{e_{33}}}{j\omega l}(v_2 - v_1), \quad (10)$$

where V is input voltage, l is total length of the transducer and v_1, v_2 are tip velocity of the transducer. Current I is calculated from D_3 as equation (11):

$$I = \frac{d}{dt}\int_S D_3 \, dS = j\omega S D_3 \quad (11)$$

3. Measurement

In this research, admittance curve was measured and used to obtain higher order elastic constant. Hard-type PZT (Fuji-ceramics C203) transducers were measured. The dimensions of the 33 effect transducer was 2 mm × 3 mm × 10 mm and polarization direction was aligned to 10 mm direction and 31 effect transducer's dimension was 7 mm × 2 mm × 44 mm, which was polarized along 2 mm direction. Frequency response analyzer (NF FRA5097) and power amplifier (NF 4010) were used to measure the admittance curve under various voltages, which are 0.25 Vpp, 10 Vpp, 40 Vpp, 70 Vpp, 100 Vpp, 200 Vpp and 300 Vpp, to measure the stress dependency of $c_{33(3)}^E$ and $c_{33(5)}^E$. From measured results, curve fitting was conducted using the nonlinear transfer matrix model for obtaining $c_{33(3)}^E$ and $c_{33(5)}^E$ under each voltages.

Figure 1 shows the relationship between real and imaginary part of $c_{11(3)}^E$ and maximum stress T_{m} obtained from 31 effect measurement. Real part of $c_{11(3)}^E$ increase and imaginary part of $c_{11(3)}^E$ decrease with increasing T_{m}. Figure 2 shows the relationship between real and imaginary part of $c_{33(3)}^E$ and T_{m} obtained from 33 effect measurement. In the fitting process of $c_{33(3)}^E$, we used two calculation models. One was 5th order model introduced in section 2 and the another was 3rd order model which fixed $c_{33(5)}^E = 0$ and only 3rd order elasticity was taken into calculation. Using 3rd order model, imaginary part of $c_{33(3)}^E$ had negative value and 5th order model calculation couldn't converge under 20.4 MPa. It is because of insufficient stress to evaluate higher order elasticity. In the range over 30 MPa, with 3rd order model, the change rate of real part and imaginary part of $c_{33(3)}^E$ were 21% and 66%, respectively. On the other hand, With 5th order model, the change rate were only 0.5% and 39 %. Figure 3 shows the relationship between real and imaginary part of $c_{33(5)}^E$ and T_{m} obtained by 5th order model fitting. Its real part increase and imaginary part decrease with increase of T_{m}. These results indicate the influence of 5th order elasticity becomes significant in high stress condition. In addition, 7th and greater order elasticity can affect under larger stress range.

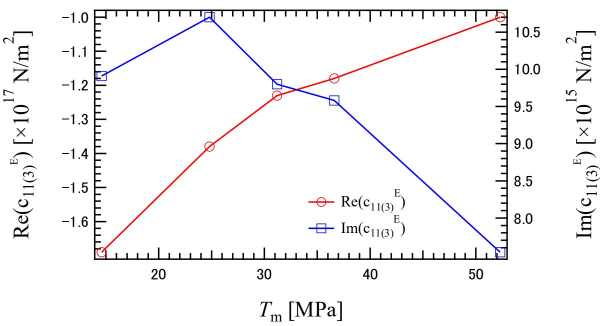

Fig. 1 The relationship between real and imaginary part of $c_{11(3)}^E$ and maximum stress T_{m} on 31 effect

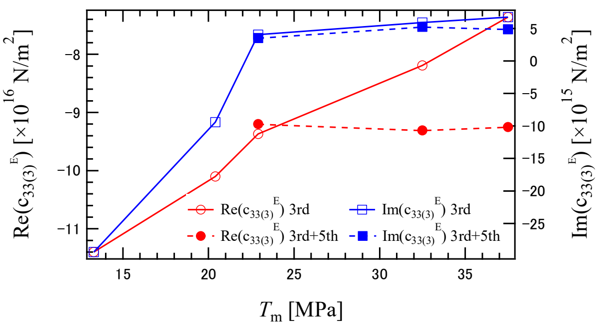

Fig. 2 The relationship between real and imaginary part of $c_{33(3)}^E$ and maximum stress T_{m} on 33 effect

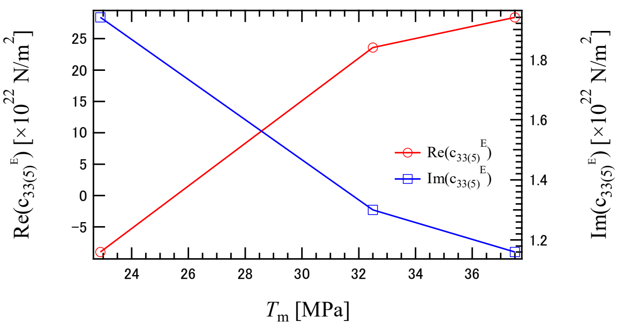

Fig. 3 The relationship between real and imaginary part of $c_{33(5)}^E$ and maximum stress T_{m} on 33 effect

4. Conclusion

In this research, 3rd and 5th order elastic constant of PZT 31 effect and 33 effect transducer were evaluated. Under high stress over 30 MPa, 5th and greater order elasticity had significant influence.

References

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