

# Ultrasound imaging with L1 norm minimization

## L1 ノルム最小化を用いた超音波撮像

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### 1. Introduction

Since three-dimensional ultrasound imaging is performed using a two-dimensional array in which ultrasound elements are arranged two-dimensionally, the number of elements used is considerably larger than that of a two-dimensional ultrasound image using a linear array. This causes problems such as an increase in manufacturing cost, an increase in difficulty in creating an array, and an increase in calculation cost for image reproduction. In general, there is a trade-off between the number of elements and the image quality, so the number of elements cannot be reduced unnecessarily. Therefore, a method for obtaining a high-quality ultrasound image with a small number of elements is required.

In recent years, compressed sensing, a method that can restore the original signal even from data with an insufficient number of samples, has attracted attention. This method pays attention to the sparsity of the data to be estimated, so that the signal can be reproduced without satisfying the sampling theorem. [1][2][3]

We verified 3D ultrasound imaging by actual measurement using L1 norm minimization, which is the main solution of compressed sensing.

### 2. Ultrasound imaging method

#### 2.1 Mathematical model of ultrasound imaging

For a certain frequency, when the frequency component (complex amplitude) of the received wave obtained by transmitting and receiving ultrasound waves is taken as the reflectance distribution, it can be expressed as Eq. (1) by modeling with a transfer matrix. can.

$$\mathbf{r} = \mathbf{H}\mathbf{s} \quad (1)$$

#### 2.2 L1 norm minimization

L1 norm minimization is a solution mainly used in the signal restoration process in compressed sensing. In this study, the reflectance distribution was calculated by Eq. (2).

$$\min_{\mathbf{s}} \left\{ \|\mathbf{s}\|_1 + \frac{1}{2\rho} \|\mathbf{H}\mathbf{s} - \mathbf{r}\|_2^2 \right\} \quad (2)$$

Here,  $\rho (= 10^{-3})$  is a Lagrange undetermined multiplier and has a role of adjusting the specific densities of the first and second terms.[4][5][6]

### 3 Data collection system for actual measurement

Figure 1 shows a schematic diagram of the data collection system for actual measurement. Data acquisition is a short pulse wave generated from a pulsar receiver, which drives a transmitting transducer to generate ultrasonic waves and receives them by a receiving transducer. The received wave is converted into received data by the data collection unit and imaged by a computer.

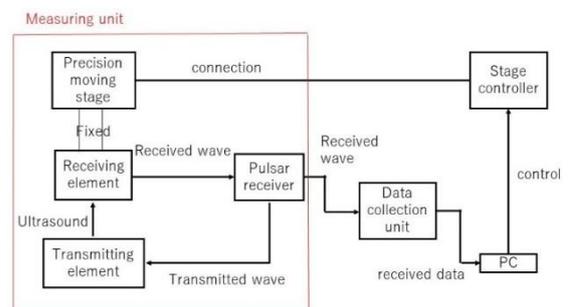


Fig.1 Overview of data collection system

### 4 Experiment

#### 4.1 Experimental conditions

The actual measurement was performed using a point object as an imaging target. Originally, ultrasound imaging transmits

ultrasound waves and receives echoes from an object, but in this study, by arranging the transmitting element at the position of the object, it was measured as a point object. The transfer matrix  $H$  of Equation 1 is required to restore the ultrasonic image from the received data. However, if all the data of the transfer matrix  $H$  is obtained from the actual measurement, it is necessary to calculate the number of actual measurements of (number of receiving elements)  $\times$  (number of field divisions), which takes time. Therefore, we Therefore was measured at several points, necessary data not actually measured was calculated using interpolation. Then, the transfer matrix  $H$  was designed based on the measured data and the data obtained by interpolation.

**Table.1** shows the parameters actually measured to calculate the transfer matrix  $H$ , and **Table.2** shows the parameters to restore the ultrasonic image. Due to the relationship between the number of duals present in this experiment and the number of divisions of the visual field, the transmission piece is 256 x 6400 pieces. Of these, 25 were obtained from actual measurements. Other elements are obtained by interpolation.

Table.1 Transfer matrix  $H$  measured parameters

Center frequency	2.5MHz
Sampling frequency	50MHz
Scanning range	20mm
Number of representative points	25
distance	150mm

Table.2 Ultrasound image restoration parameters

Center frequency	2.5MHz
Sampling frequency	50MHz
Receive array dimensions	10mm
Number of receiving elements	256
Number of transmitting elements	1
distance	150mm
Number of field divisions	80*80

## 4.2 Experimental result

**Figure 2** shows a volume-rendered image of the point object restored by the experiment.

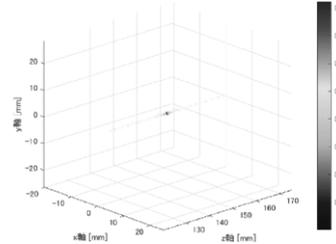


Fig.2 Volume rendered image of a point object

From Fig.2, there is a pixel at a distance of 150 mm, and the spread is small, so it can be judged as a point object.

## 5 Conclusion

It was difficult to recognize even a point object by the actual measurement of ultrasonic imaging of the transfer matrix obtained only by calculation. Ultrasound imaging using a transfer matrix designed based on some actually measured data and data obtained by interpolation succeeded in reconstructing a point object. At this time, it was found that when reconstruction was performed using the principle of L1 norm minimization, a high-resolution image with fewer virtual images could be obtained as compared with the conventional method.

## References

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