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Elasticity measurement of radial artery wall considering the change in cross-sectional shape of vessel caused by pushing pressure from ultrasound probe

超音波プローブの押圧による血管断面形状の変化を考慮した橈骨動脈壁弾性計測

Yuto Shoji[1‡], Shohei Mori[2], Mototaka Arakawa[1,2], Shigeo Ohba[2], Kazuto Kobayashi[3], and Hiroshi Kanai[2,1] ([1]Grad. School Biomed. Eng., Tohoku Univ.; [2]Grad. School Eng., Tohoku Univ.; [3]Div. Res., Honda Electron. Co. Ltd.)

庄司悠人 [1†]，森翔平 [2]，荒川元孝 [1,2]，大庭茂男 [2]，小林和人 [3]，金井浩 [2,1] ([1]東北大院・医工，[2]東北大院・工，[3]本多電子(株)研究部)

1. Introduction

In the early stages of atherosclerosis, vascular endothelial dysfunction occurs. We have developed an ultrasound probe that can simultaneously measure the blood pressure and the vessel diameter, to evaluate endothelial function by measuring viscoelasticity during the flow-mediated dilatation (FMD) [1]. As the blood pressure measurement with the probe requires deforming the blood vessel, it is necessary to consider the effect of the change in vessel shape for the viscoelasticity estimation [2].

In the present study, we derived a series of equations to estimate the elastic modulus considering the change in the cross-sectional shape of the blood vessel from the circle due to the pushing pressure by the ultrasound probe, and examined the validity of the equation by numerical calculations using the parameters in the actual measurements.

2. Method

First, the circumferential and axial incremental strains, $\Delta\varepsilon_\theta$ and $\Delta\varepsilon_z$, of a homogeneous isotropic material are respectively expressed in the polar coordinate system by the following equations: [3]

$$\Delta\varepsilon_\theta = \frac{\Delta\sigma_\theta}{E} - \frac{\nu\Delta\sigma_r}{E} - \frac{\nu\Delta\sigma_z}{E}, \quad (1)$$

$$\Delta\varepsilon_z = \frac{\Delta\sigma_z}{E} - \frac{\nu\Delta\sigma_r}{E} - \frac{\nu\Delta\sigma_\theta}{E}, \quad (2)$$

where $\Delta\sigma_r$, $\Delta\sigma_\theta$ and $\Delta\sigma_z$ are the incremental stresses in the radial, circumferential, and axial directions, respectively, and E is the elastic modulus, and ν is the Poisson's ratio. As the blood vessels are axially constrained *in vivo*, the $\Delta\varepsilon_z$ can be assumed to be negligible. The arterial wall is assumed to be incompressible. Then, the following equation is obtained from Eq. (2).

$$\Delta\sigma_z = \frac{1}{2}(\Delta\sigma_r + \Delta\sigma_\theta). \quad (3)$$

Next, we consider the balance of forces when the radial artery is deformed by the internal pressure due to the heartbeat and the pushing pressure due to the ultrasound probe. The cross-section of the artery

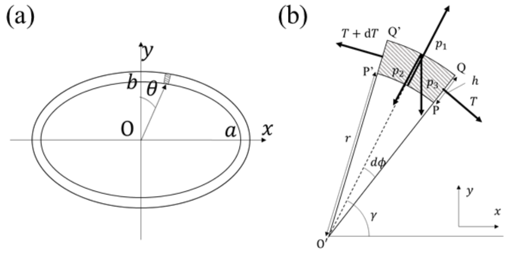

Fig. 1. The balance of forces in the vascular wall. (a) Schematic diagram of cross section of the blood vessel, (b) the magnified view of the micro-region of blood vessel wall.

is assumed to be deformed from the circle to the ellipse as shown in **Fig. 1**(a) due to the uniform pushing pressure applied on the upper surface along the *y*-direction. Figure 1(b) shows the magnified view in the shaded region in Fig. 1(a). In Fig. 1(b), p_1 is the intravascular pressure, p_2 is the atmospheric pressure, and p_3 is the pushing pressure by the ultrasound probe. Moreover, T is the tension acting in the circumferential direction of the vessel, r is the curvature radius in the micro-region of the ellipse, and h is the thickness of the vessel wall. We assumed that h does not change during the deformation from the circle to the ellipse and its uniformity within the micro-region.

Let us consider the relationship between the change $\mathrm{d}T$ in T within the micro-region and the balance with the force in the *x* and *y* directions acting on the micro-region. By eliminating $\mathrm{d}T$ from the equations in the two directions and ignoring the terms larger than the square of the small amount, T is given by

$$T = r\left(p_1 - p_2 - \frac{\tan^2\gamma}{1+\tan^2\gamma}p_3\right) - h\left(p_2 + \frac{\tan^2\gamma}{1+\tan^2\gamma}p_3\right). \quad (4)$$

If σ_θ is constant along r direction, σ_θ is obtained by dividing both sides of Eq. (4) by h as

y.shoji@ecei.tohoku.ac.jp

$$\sigma_\theta = \frac{1}{abh}\left(\frac{a^2 + b^2\tan^2\theta}{1+\tan^2\theta}\right)^{\frac{3}{2}}\left(p_1 - p_2 - \frac{a^2\tan^2\theta}{a^2 + b^2\tan^2\theta}p_3\right) - \frac{a^2\tan^2\theta}{a^2 + b^2\tan^2\theta}p_3, \quad (5)$$

where a, b, and θ are shown in Fig. 1(a). On the other hand, the radial stress σ_r is obtained by the balance of the radial partial forces of p_1 in the lumen, and p_2 and p_3 in the outer surface as follows

$$\sigma_r = -\frac{1}{2}\left(p_1 + p_2 + \frac{a\tan\theta}{\sqrt{a^2\tan^2\theta + b^2}}p_3\right). \quad (6)$$

By substituting $\Delta\varepsilon_\theta$ and the incremental stresses $\Delta\sigma_\theta$ calculated from Eqs. (3), (5), and (6) into Eq. (1), elastic modulus E can be estimated, where $\Delta\varepsilon_\theta$ is calculated from the change in the vessel diameter measured by ultrasound.

3. Experiment

We numerically obtained σ_θ to verify whether the derived stress σ_θ represents the actual movement of the blood vessel during the elasticity measurement. We substituted p_2 = 1013.25 hPa, p_3 = 150 mmHg, a = 1.2 mm, b = 0.7 mm, and h = 0.43 mm into Eq. (5), and varied θ from 0° to 180°. σ_θ was obtained by substituting p_1 = 120 and 72 mmHg in systole and diastole, respectively. The incremental stresses from diastole to systole were also calculated. As it is difficult to measure h in the radial artery *in vivo*, it was set to 0.43 mm in literature [4].

To examine the relationship between σ_θ and the vessel shape, σ_θ was obtained by varying the aspect ratio of the cross-section of the vessel a/b from 1 to 3. In addition, to examine the relationship between p_3 and σ_θ, σ_θ was obtained by varying p_3 from 0 to 150 mmHg. When calculating the shape and the pushing pressure dependence, σ_θ was obtained at $\theta = 0°$ because the ultrasound beam which passes through the center of the vessel was used for the strain measurement.

4. Result and Discussion

Figure 2(a) shows the calculated results for the θ dependence of σ_θ using Eq. (5) at systole, diastole, and the increment from diastole to systole by red, blue, and green lines, respectively. Figure 2(a) shows the minimum and maximum values at $\theta = 0°$ and 90° for both systole and diastole, respectively, where the negative (positive) stress compresses (elongates) the vessel wall. When the cross-section of the blood vessel was deformed from the circle to the ellipse by the pushing pressure, the blood vessel wall elongates circumferentially in the short axis and contracts circumferentially in the long axis because the stress exits to balance the force. The incremental stress became maximum in the short axis (θ=0°) and minimum in the long axis (θ=90°). Since the displacement in the short axis is larger than that in the long axis when the internal pressure is applied to the ellipse [5], this result is reasonable.

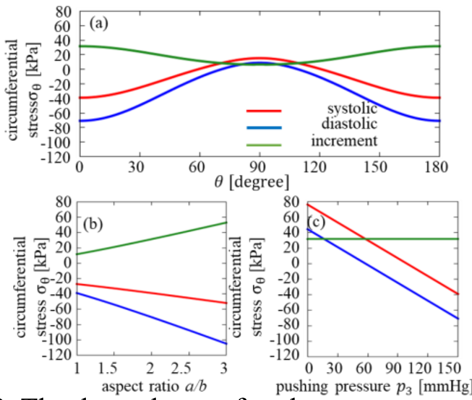

Fig. 2. The dependence of each parameter on σ_θ. (a) θ, (b) a/b, (c) p_3.

Figs. 2(b) and 2(c) show the calculated results for the shape and pushing pressure dependences of σ_θ, respectively. From Fig. 2(b), as the aspect ratio a/b increased, that is, the cross-section of the vessel deformed, σ_θ decreased but the incremental stress increased. As the deformation of the cross-section of the vessel increased, the maximum strain of the vessel increased during one heartbeat. Therefore, this result is also reasonable.

As shown in Fig. 2(c), the incremental stress did not change while σ_θ decreased as p_3 increased. As the pushing pressure p_3 increased, the blood vessel deformed and the stress increased in the compressive direction to balance this. On the other hand, the incremental stress was constant if the pushing pressure was constant during one heartbeat. Therefore, we consider that the elasticity measurement at rest was stable in the previous study [6].

5. Conclusion

For the elasticity measurement of the radial artery vessel wall using a single ultrasonic probe, the equations to estimate the elastic modulus considering the pushing pressure by the ultrasonic probe and the shape change of the vessel were derived. From the numerical calculations, it was confirmed that the trend of the variation was consistent with the results of the previous measurements. We plan to apply these equations to actual elastic modulus measurements in the future.

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