

# Estimation of charges of a sonoluminescing bubble under electric field

## ソノルミネセンス気泡の電荷量評価

Hyang-Bok Lee<sup>1†</sup>, Pak-Kon Choi<sup>2</sup>, Kageyama Yuya<sup>2</sup>  
(<sup>1</sup>Japan Women's Univ.; <sup>2</sup>Meiji Univ.)

李 香福<sup>1†</sup>, 崔 博坤<sup>2</sup>, 影山 祐弥<sup>2</sup> (<sup>1</sup>日女大理, <sup>2</sup>明大理工)

### 1. Introduction

A bubble undergoing the violent oscillation of contraction and expansion emits light when the bubble collapses at the antinode of standing wave. This phenomenon is known as single bubble sonoluminescence (SBSL). Since SBSL bubble is spatiotemporal stable, SBSL bubble has been a subject of research to elucidate the dynamics of acoustic cavitation bubbles. Recently, Lee et al.[1] experimentally showed that the SBSL bubble in water was positively charged from the translation of the bubble when external electric field applied to the bubble. Under electric field, the SBSL bubble shifted to equilibrium position balancing the Bjerknes force and the electrostatic force as shown in **Figure 1**. However, the evaluation of the amount of charges of the SBSL bubble remained.

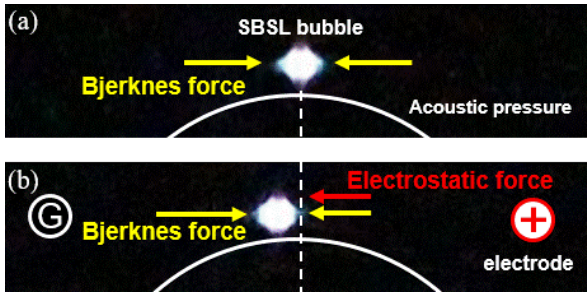


Fig. 1 Translation of SBSL bubble by electrostatic force (b). White dotted line indicate the initial bubble position in the absence of an electric field (a)

### 2. Methods

The amount of charges of SBSL bubble can be evaluated from the magnitude of Bjerknes force which is radiation force acting on bubble from ultrasound. It is necessary to know the acoustic pressure and initial bubble radius to obtain the Bjerknes force. It is difficult, however, to obtain them experimentally. Therefore, we obtained the initial bubble radius by calculation using Keller's equation, which expresses the bubble oscillations of expansion and contraction. By comparing these calculation results with the bubble radius obtained by light scattering technique [1], we obtained the initial

radius and acoustic pressure required for the calculation of Bjerknes force.

Bubble radius was calculated using the Keller's equation given by the following.

$$\left(1 - \frac{\dot{R}}{c}\right) R \ddot{R} + \frac{3\dot{R}^2}{2} \left(1 - \frac{\dot{R}}{3c}\right) = \frac{1}{\rho} \left(1 + \frac{\dot{R}}{c}\right) \left[ P_B - P_a \left(t + \frac{R}{c}\right) - P_0 \right] + \frac{R}{c\rho} \frac{dP_B}{dt},$$

where

$$P_B = \left( P_0 + \frac{2\sigma}{R_0} - P_v \right) \left( \frac{R_0}{R} \right)^{3\gamma} + P_v - \frac{2\sigma}{R} - \frac{4\mu\dot{R}}{R} \quad (1)$$

where  $P_a$  is the acoustic pressure,  $P_0$  is the hydrostatic pressure,  $R_0$  is the initial bubble radius,  $P_v$  is the vapour pressure,  $\gamma$  is polytropic index of the gas within the bubble,  $\sigma$  and  $\mu$  is the liquid surface tension and viscosity respectively. The parameters used for the calculation were selected from the measurement conditions of Lee et al. [1]. The ultrasound frequency is 28.45 kHz and the liquid temperature is 13 °C. We assume that the gas within bubble is argon. The shape of the expansion and contraction of bubble is similar with different initial bubble radius of in different acoustic pressures. Hence, the bubble radius was calculated as a function of the initial radius of 2-10  $\mu\text{m}$  in the acoustic pressure range of 1.4-1.65 atm realized in the SBSL experiment. By comparing these calculation results with the bubble radius obtained from the light scattering technique, the initial radius and acoustic pressure required for the Bjerknes force calculation were obtained.

Acoustic radiation force acting on bubble in standing waves, known as Bjerknes force, is given by [2,3]

$$F_{\text{Bjerk}} = \langle -V\nabla P \rangle = \langle F_p \rangle \quad (2)$$

$$F_p = -\frac{4\pi R^3}{3} k P_a \sin(kx) \sin(\omega t) \quad (3)$$

where the  $V$  is bubble volume,  $\nabla P$  is the gradient of the applied acoustic pressure,  $F_p$  is the

instantaneous force on the bubble and  $\langle \dots \rangle$  denotes time average. From the results of Lee et al. [1], the bubble position was shifted about  $82\mu\text{m}$  from the antinode of the standing wave when a voltage of  $+50\text{V}$  was applied to the electrode. The instantaneous force  $F_p$  was calculated where the bubble is located  $82\mu\text{m}$  ( $x = 82\mu\text{m}$ ) from the antinode of the standing wave in the radial direction.  $k$  is the wave number of the standing waves formed in the cylindrical cell as expressed in equation (5) in ref.[1].

The bubble position is equilibrium position balancing the Bjerknes force and electrostatic force under electric field. From the calculated result of the Bjerknes force and the electric field strength at the bubble position, the amount of charges of the bubble was estimated from the following equation.

$$F_{\text{Bjerk}} = qE \quad (4)$$

### 3. Results and discussion

**Figure 2** shows the bubble radius where the calculation result optimally agrees with that obtained by light scattering technique. The dashed line indicates the calculated value for initial bubble radius of  $3.2\mu\text{m}$  and acoustic pressure of  $1.50\text{ atm}$ . The solid line indicates the bubble radius obtained by light scattering technique. Those values are normalized to their maxima. The calculated data and measured data are in good agreement with the expansion and contraction of bubble and the rebound after the bubble collapses. From this result, the initial radius of  $3.2\mu\text{m}$  and the acoustic pressure of  $1.50\text{ atm}$  were used to calculate the Bjerknes force.

The instantaneous force on the bubble was

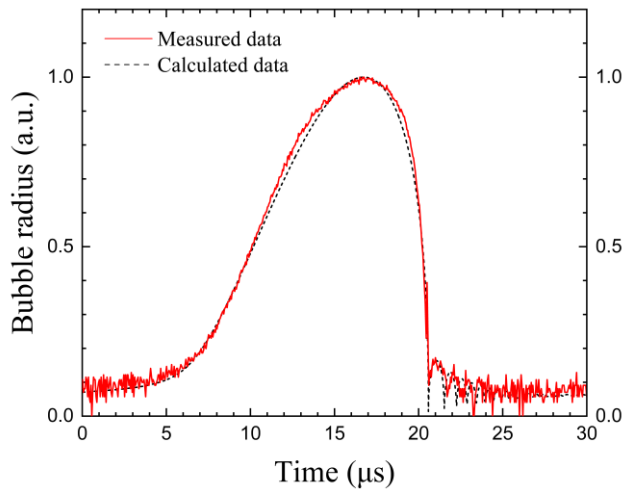


Fig. 2 Comparison of bubble radius between the calculated value using Eq. (1) (dashed line) and the experimental value (solid line).

calculated using eq. (3) with parameters obtained in the above calculation. **Figure 3** shows the results. The solid line (a) is the bubble radius calculated using eq. (1). The dashed line (b) is the instantaneous force on the bubble located at  $82\mu\text{m}$  in the radial direction from the antinode of the standing wave. The dotted line (c) is the acoustic pressure. During the negative portion of the instantaneous force, the bubble towards the antinode of the standing wave. Conversely, during the positive portion of the instantaneous pressure force, the bubble towards the node of the standing wave.

The Bjerknes force obtained by averaging the force in one cycle was  $-1.27 \times 10^{-9}\text{ N}$ . Amount of charge of bubble under electric field can be calculated from the balance between the Bjerknes force and the electrostatic force as expressed in eq. (4). Electric field strength was measured to be  $500\text{ V/m}$  within the region  $\pm 1\text{ mm}$  around the initial bubble position. The amount of charge of the bubble under electric field was estimated to be  $2.5 \pm 0.3\text{ pC}$ . This is the first study to estimate the charges of SBSL bubble.

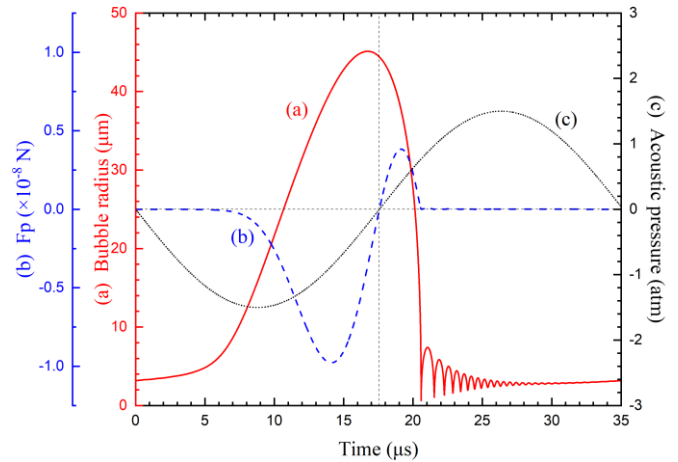


Fig. 3 Instantaneous Bjerknes force on the bubble calculated using Eq. (3) (dashed line). Solid line (a) and dotted line (c) indicate bubble radius and the time curve of acoustic pressure, respectively.

### Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP19K15452.

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