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Numerical analysis of ultrasonic wave propagating through a thin plate stacked structure
薄板積層構造を透過する超音波の数値解析

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1. Introduction

Metamaterials are substances that are artificially created so that they have properties that natural substances do not normally have. Metamaterials for sound waves are called acoustic metamaterials, and research is being conducted to develop structures that control the propagation of sound waves, such as acoustic diodes[1] and acoustic cloaking[2]. Acoustic metamaterials capable of such direction control are expected to be applied to, for example, energy harvesting that collects environmental vibrations to generate electricity. In previous studies of acoustic metamaterials, the structure is often determined exploratorily resulting in a complex and poorly versatile structure.

Therefore, the purpose of this study is to control the direction of transmitted waves after analyzing the Lamb wave propagation behavior in a relatively simple thin plate structure.

2. Wave propagation in media with periodic boundary conditions

As shown in **Fig. 1**, when the periodic structure in y direction is virtually considered. and the same vibration load is applied to each periodic structure, the wave field also becomes periodic.

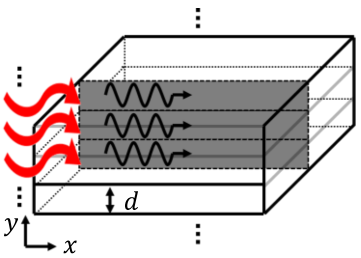

Fig. 1　Wave propagation in periodic structure

Considering wave propagation in a periodic structure for each width d in the y direction, the phases are necessary to match at positions separated by a distance d in the y direction. When transverse waves propagate in this structure, the wavenumbers in the x and y directions, k_{β_n} and β_n, respectively, satisfy the following equations

$$k_{\beta_n} = \pm\sqrt{(\omega/c_T)^2 - (2n\pi/d)^2},\quad \beta_n = 2n\pi/d \qquad (1)$$

where ω is angular frequency, c_T is transverse wave velocity, and $n = 0, 1, 2, \ldots$.

3. Periodically plate-stacked structure

The dispersion curve of Lamb wave A0 mode is shown in **Fig. 2**, where horizontal and vertical axes are the product of frequency and plate thickness (*fd*) and the phase velocity, respectively. Since the phase velocity changes significantly in the low *fd* region, the phase velocity at the fixed frequency changes greatly depending on the plate thickness.

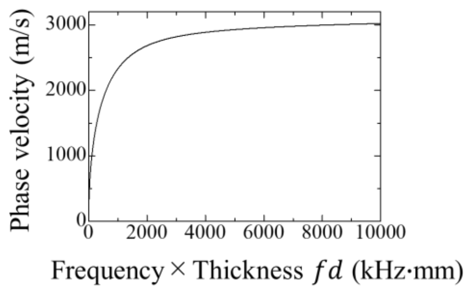

Fig. 2　Phase velocity dispersion curve (Lamb wave A0 mode)

As an example, consider a structure having a plate-stacked structure of length L in which thin plates of different thickness are stacked as shown in **Fig. 3**. When a transverse plane wave with a uniform distribution of vibration in the vertical direction is incident, it branches in a stacked structure and propagates in each layer as a Lamb wave A0 mode at different velocities. When the Lamb waves that have branched and propagated for the distance L merge, the phases of the components from each layer are different. The phase shift generates new wave fields in the right media. When the stacked structure as shown in **Fig. 3** is stacked periodically, the transmitted transverse wave is constrained by the conditions shown in Eq. (1). Therefore, when the

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phase shift due to the stacked structure is matched to the phase distribution $\beta_n y$ in the y direction, a plane wave of a certain order n is transmitted.

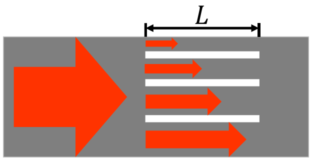

Fig. 3 Branching of incident waves due to plate-stacked structure

4. Numerical simulation

To confirm the plane wave formed by the phase shifted Lamb waves in a plate-stacked structure, a numerical simulation was performed using COMSOL Multiphysics for the structure shown in **Fig. 4**. A traction free boundary was set for the plate-stacked structure. The upper and lower boundaries were set as periodic boundaries, and calculations were performed for one layered region of the periodic structure. Absorption layers[3] were set at the left and right ends so that analysis can be performed without reflected waves. Vertical body force was applied to all cells located 2 mm from the left edge of the material region to excite the transverse wave.

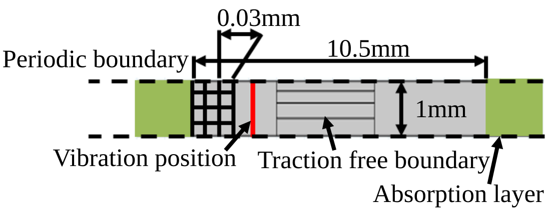

Fig. 4 Analytical structure

First, consider a structure that allows a transverse plane wave with $n = 0$ to pass through. The wavenumbers of the transverse wave with $n = 0$ in the x and y directions are expressed as

$$k_{\beta_0} = \omega / c_T, \qquad \beta_0 = 0 \tag{2}$$

by Eq. (1), respectively. This means that a transverse plane wave with $n = 0$ propagates in a direction parallel to the periodic boundary. The numerical results for a four-layer periodic thin plate stacked structure with the same layer thickness are shown below. **Fig. 5** shows an image that visualizes the transverse wave component of the displacement in the steady state when vibration is applied by a harmonic wave of 4.5 MHz. Four cycles of the periodic structure are displayed in color, with red and blue parts being the belly and yellow green being the nodes. A stacked structure is set in the part surrounded by the black line. A transverse plane wave with $n = 0$ is transmitted.

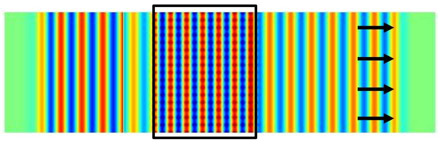

Fig. 5 Wave propagation through a structure consisting of plates with equal thickness

Next, consider a transverse plane wave with $n \geq 1$. This case, from Eq. (1), indicates a transverse plane wave propagating in a direction diagonal to the periodic boundary. The length of the plate-stacked structure and the thickness of each layer were adjusted so that the refracted plane wave of $n = 1$ could be transmitted, and the plate-stacked structure of 4 layers was designed. **Fig. 6** shows an image that visualizes the transverse wave component of displacement in the steady state.

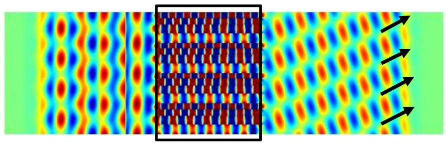

Fig. 6 Wave propagation through a plate-stacked structure designed for refracted wave of $n = 1$

This figure clearly shows that the wavefront after transmission was slanted through the stacked structure surrounded by the black line.

5. Conclusion

To control the direction of transmitted waves using plate-stacked structures, numerical analysis of Lamb waves transmitted through a periodic plate-stacked structure was performed.

When transverse waves are incident from the end faces in a plate-stacked structure with the same layer thickness, there is no phase difference between the Lamb waves propagating in each layer, so the transmitted wave was a transverse plane wave with $n = 0$ propagating parallel to the periodic boundary.

When the thickness of the plate-stacked structure was adjusted to meet the phase shift with the transverse plane wave of $n = 1$ propagating diagonally with respect to the periodic boundary, the refracted plane wave was transmitted as expected.

References

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2. M. Maldovan, *Nature*. **503**, 209 (2013).
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