3Pa5-3

Design of robust broadband filter based on truncated singular value decomposition for ultrasound received signal matrix
超音波受信信号に対する打ち切り特異値分解に基づくロバストな広帯域化フィルタの設計

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1. Introduction

We are aiming to observe biological tissues with a high spatial resolution by using a filter to widen the frequency band in the ultrasonic measurement [1]. In the present study, we designed a noise-robust filter using truncated singular value decomposition (SVD) and proposed a reasonable procedure to determine the truncated order of singular values.

2. Principles

2.1 Filter design based on truncated SVD

It is assumed that a point spread function can be approximated by the radiofrequency (RF) signal received from a thin wire. A filter is designed to transfer the point spread function to the impulse. First, we prepare a $(2N+1)\times(2M+1)$ matrix $\mathbf{Y_0}$ composed of RF signals received from the thin wire, $\{y(l)\}$, whose sample points are shifted one by one as shown in **Fig. 1(b)**, where l is a sample point index. The $(i,\ j)$ element of $\mathbf{Y_0}$ is represented as

$$\mathrm{Y}_0(i,j) = y(p-M-N+i-j), \qquad (1)$$

where p is the positive peak point of $y(l)$. We also prepare a $(2N+1)$-dimensional model vector $\mathbf{m}$ of impulse as defined by

$$\mathrm{m}(i) = \begin{cases} 1 & (i = N+1) \\ 0 & (i \neq N+1) \end{cases}. \qquad (2)$$

The filter $\mathbf{f}_R$ is designed to minimize the squared error $\alpha(R)$ between $\mathbf{Y}_0\mathbf{f}_R$ (the output after applying the filter) and $\mathbf{m}$ (the impulse model) as

$$\alpha(R) = |\mathbf{Y}_0\mathbf{f}_R - \mathbf{m}|^2. \qquad (3)$$

The filter $\hat{\mathbf{f}}_R$ which minimizes $\alpha(R)$ is estimated from the least-squares method as

$$\hat{\mathbf{f}}_R = \mathbf{V}\mathbf{\Sigma}_R^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{m}, \qquad (4)$$

where $\mathbf{U}$ and $\mathbf{V}$ are the left and right singular vector matrices in the SVD of $\mathbf{Y}_0$, and are unitary matrices with sizes of $2N+1$ or $2M+1$, respectively. $\mathbf{\Sigma}_R^{-1}$ is a $(2M+1)\times(2N+1)$ matrix that takes the reciprocal of singular values in

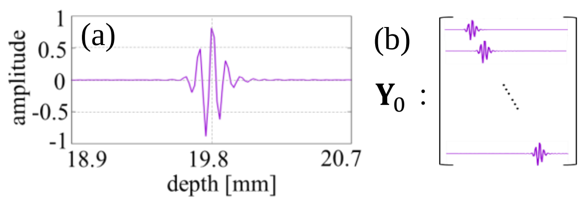

Fig. 1 (a) Received RF signal from a thin wire and (b) conceptual diagram of $\mathbf{Y_0}$.

diagonal elements and is truncated at order R as

$$\Sigma_R^{-1}(i,j) = \begin{cases} 1/\sigma_{i,j} & (1 \leq i = j \leq R) \\ 0 & (R+1 \leq i = j \leq 2M+1) \\ 0 & (i \neq j) \end{cases}, \qquad (5)$$

where $\{\sigma_{i,j}\}$ $(\sigma_{1,1} \geq \sigma_{2,2} \geq \cdots \geq \sigma_{2M+1,2M+1})$ are singular values. Thus, the resultant filter $\hat{\mathbf{f}}_R$ can suppress the components higher than the truncated order R and emphasize only the components lower than R. We obtain the output RF signal $y_{\mathrm{out}}(l)$ by convolving the filter $\hat{\mathbf{f}}_R$ in the time domain as

$$y_{\mathrm{out}}(l) = \sum_{i=0}^{2M} y(l-M+i)\cdot \hat{\mathrm{f}}_R(i), \qquad (6)$$

where $\hat{\mathrm{f}}_R(i)$ shows the i-th element of $\hat{\mathbf{f}}_R$.

2.2 Determination of the truncated order of singular values for the noise-robust filtering

Since the lower-order components of the singular values are dominated by the signal components and the higher-order components are dominated by noise, it is necessary to truncate singular values at an appropriate order R for the noise-robust filtering. Therefore, we propose the following determination method of the truncated order of singular values.

At first, white noise $n_k(l)$ is generated independently in K trials and is superimposed on the signal $y(l)$ received from the thin wire as

$$y_{\mathrm{noisy},k}(l) = y(l) + n_k(l). \qquad (7)$$

We define $\mathbf{Y}_{\mathrm{noisy},k}$ as the matrix obtained by substituting $y_{\mathrm{noisy},k}(l)$ into $y(l)$ in Eq. (1). The truncated order $\hat{R}$ is determined to minimize $\beta(R)$.

$$\hat{R} = \arg\min_R \beta(R), \qquad (8)$$

$$\beta(R) = \mathrm{E}_k\left[\left|\mathbf{Y}_{\mathrm{noisy},p}\hat{\mathbf{f}}_R - \mathbf{m}\right|^2\right], \qquad (9)$$

where $\mathrm{E}_k[\cdot]$ denotes the average operation about $0 < k \leq K$. By determining R to minimize $\beta(R)$, the filter $\hat{\mathbf{f}}_{\hat{R}}$ can be optimized considering the trade-off relationship between the spatial resolution improvement and the noise robustness.

3. Method

An optimal filter $\hat{\mathbf{f}}_{\hat{R}}$ was designed based on Eqs. (1)-(9) using the RF signal $y(l)$ received from the thin wire with a 10-μm diameter in water. The Gaussian white noise $n_k(l)$ was generated on the computer and superimposed on $y(l)$. The signal-to-

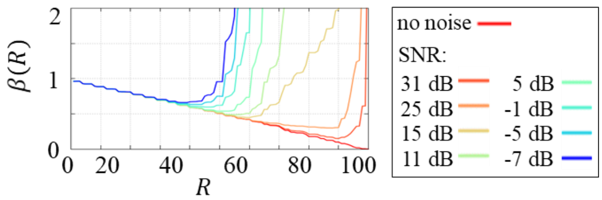

Fig. 2 $\beta(R)$ calculated for each SN ratio.

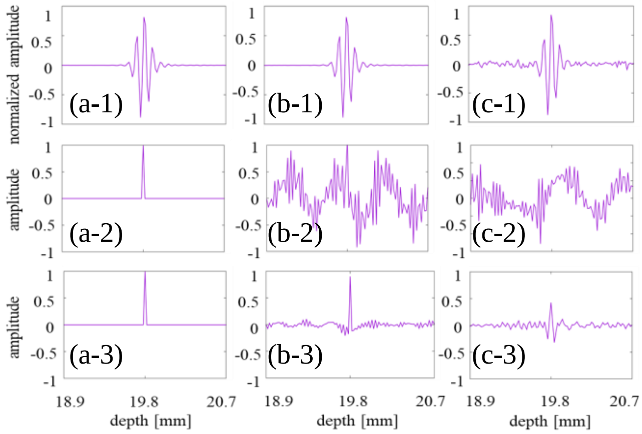

Fig. 3 (1) RF signal $y_{\mathrm{noisy},k}(l)$ before applying the filter. RF signal $y_{\mathrm{out}}(l)$ after applying the filter with (2) $R = 101$ (full rank) and (3) $R = \hat{R}$ (optimal truncated order). SN ratios were set to (a) ∞ (no noise), (b) 25 dB, and (c) -5 dB.

noise ratio (SNR) was defined as the ratio of the averaged power of the received RF signal $y(l)$ from the wire to the averaged noise power. The SNR was set from -7 dB to 31 dB and the optimal truncated order $\hat{R}$ was determined for each SNR, independently. The number of trials K for calculation of Eq. (9) was set to 200, and the size of the received RF signal matrix $\mathbf{Y}_0$ was set to $2M + 1 = 2N + 1 = 101$.

The optimal filter $\hat{\mathbf{f}}_{\hat{R}}$ designed by water tank experiment with the wire of 10-µm diameter was also applied to the RF signal received from a wire phantom (CIRS: Model 054GS), where two nylon wires with a diameter of 80 µm were placed 0.5 mm apart in the beam direction. A linear probe was used for the measurement with a center frequency of 7.5 MHz and a sampling frequency of 40 MHz.

4. Results

Figure 2 shows $\beta(R)$ calculated for each SNR, independently. From Fig. 2, the optimal truncated order $\hat{R}$ was shifted to the lower order as the SNR decreases. In addition, $\beta(R)$ increases rapidly at $R > \hat{R}$. These results suggest that the truncated order $\hat{R}$ should be determined depending on the noise level for the noise-robust filtering.

Figure 3 shows (1) the waveform of $y_{\mathrm{noisy},k}(l)$ before applying the filter, (2) the result $y_{\mathrm{out}}(l)$ of applying the filter $\hat{\mathbf{f}}_{R=101}$ obtained with full rank ($R = 101$), and (3) the result $y_{\mathrm{out}}(l)$ of applying the filters $\hat{\mathbf{f}}_{\hat{R}=101}$, $\hat{\mathbf{f}}_{\hat{R}=55}$, and $\hat{\mathbf{f}}_{\hat{R}=43}$ designed for SNR of ∞, 25 dB, and -5 dB, respectively. When noise was not superimposed (Fig. 3(a)), the use of the full rank of the singular value resulted in an ideal impulse. On the other hand, when noise was superimposed and the full rank of the singular value was used, the wire could not be observed (Fig. 3(b-2)). However, in Fig. 3(b-3), even when noise was superimposed, the echo from the wire could be observed by truncating the singular value at the optimal $\hat{R}$ and the spatial resolution was improved compared to that before applying the filter (Fig. 3(b-1)).

Although the spatial resolution in Fig. 3(c-3) which has a high noise level was lower than that in Fig. 3(b-3) because the optimal $\hat{R}$ was needed to be lower to suppress the high level of noise, the spatial resolution of Fig. 3(c-3) was improved compared to that before applying the filter in Fig. 3(a-3). Thus, the proposed filter could improve the spatial resolution regardless of the noise level and could optimize the trade-off relationship between the spatial resolution improvement and the noise robustness.

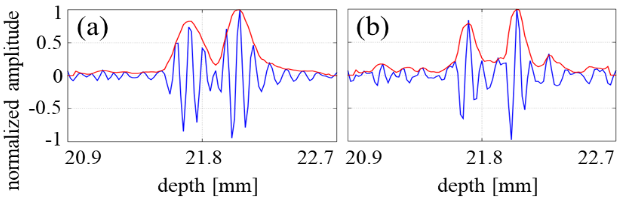

Fig. 4 RF signals (a) before and (b) after applying the filter. RF signals are received from wire phantom.

Figure 4 shows the results before and after applying the filter $\hat{\mathbf{f}}_R$ to a phantom with two wires aligned in the beam direction. The blue lines are the received RF signals and the red lines are their envelopes. After applying the filter, the SNR became lower due to the band widening effect; however, the half widths of the first and second echoes from the wires were significantly improved from 0.23 mm and 0.21 mm before applying the filter to 0.14 mm and 0.13 mm after applying the filter, respectively.

5. Conclusion

In the present study, we designed a band widening filter using truncated singular value decomposition and proposed the determination method of the truncated order of singular values for noise-robust filtering. In the future, we will design and apply the filter to *in vivo* measurements.

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References

1. K. Kawamata, S. Mori, M. Arakawa, H. Kanai, Proc. Symp. Ultrason. Electr., **41**, 3Pb5-7 (2020).