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Bayesian Filtering for Parameter Estimation of Mechanical Properties of Isotropic Material

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1. Introduction

The Lead Zirconate Titanate (PZT) is extensively used in telecommunications, actuators, energy harvesting and structural health monitoring devices[1, 2)]. These applications require knowledge regarding the dielectric, elastic and piezoelectric properties of the material used. The experimental methods such as static or dynamic methods are used to determine the elastic constants[3)]. The optical methods are employed on the studies of the photoelastic properties are based on the ultrasonic method. The ultrasonic pulse technique is employed for the determination of elastic constants[4)]. The limitation of this method is unsatisfactory accuracy for the specimen like thin plate. The elastic constants of isotropic and anisotropic solids are computed by resonant ultrasound spectroscopy technique[5)].

In this work, we propose to use the non-linear Bayesian filter to estimate the mechanical properties of the isotropic material[6)]. The Bayesian filter is a probabilistic framework that has advantages. It can provide a relaible estimated parameter. Kalman filter is one of the Bayesian filtering techniques widely used across many engineering disciplines, such as in the application of target tracking, weather and climate prediction, and spacecraft guidance[7, 8)]. Extended Kalman Filter (EKF) is used in this study amidst all the Bayesian filters available in the literature.

This paper is organised as follows: the mathematical background of the EKF is briefly described in section 2. Section 3 shows the results and discussions, while conclusions are drawn in Section 4.

2. Extended Kalman Filter

The EKF [9)] is an extension of Kalman Filter (KF) [10)] can be used for non-linear state estimation. The EKF functions around the existing KF scheme by linearising the non-linear functions. An analytical linearisation approach comprises the computation of the Jacobian matrices is used to address the non-linearity.

Consider a general discrete non-linear state-space system modelled as:

$$\begin{aligned} \mathbf{x}_{i+1} &= \mathbf{f}\left(\mathbf{x}_i, \mathbf{w}_i\right) \\ \mathbf{y}_{i+1} &= \mathbf{h}\left(\mathbf{x}_i, \mathbf{v}_i\right) \end{aligned} \tag{1}$$

where $\mathbf{f}$ is the vector-valued state prediction function and $\mathbf{h}$ is the vector-valued observation function. The $\mathbf{x}_i$ is the state variable, $\mathbf{y}_i$ is the output vector, $\mathbf{w}_i$ is the process noise with covariance $\mathbf{Q}_i$, and $\mathbf{v}_i$ is the measurement noise with covariance $\mathbf{R}_i$. The subscript i is the discrete step. The EKF operates in two steps: prediction and correction. In the prediction step, the predicted mean and error covariance is computed by:

$$\begin{aligned} \hat{\mathbf{x}}_{i+1|i} &= \mathbf{f}\left(\hat{\mathbf{x}}_i, \mathbf{0}\right) \\ \mathbf{P}_{i+1|i} &= \mathbf{F}_i \mathbf{P}_i \mathbf{F}_i^T + \mathbf{L}_i \mathbf{Q}_i \mathbf{L}_i^T \end{aligned} \tag{2}$$

In the correction step, the corrected mean and error covariance is calculated with the help of Kalman gain and is given by:

$$\begin{aligned} \mathbf{K}_{i+1} &= \mathbf{P}_{i+1|i} \mathbf{H}_{i+1}^T \left(\mathbf{H}_{i+1} \mathbf{P}_{i+1|i} \mathbf{H}_{i+1}^T + \mathbf{M}_{i+1} \mathbf{R}_{i+1} \mathbf{M}_{i+1}^T\right)^{-1} \\ \hat{\mathbf{x}}_{i+1} &= \hat{\mathbf{x}}_{i+1|i} + \mathbf{K}_{i+1}\left(\mathbf{y}_{i+1} - \mathbf{h}\left(\hat{\mathbf{x}}_{i+1|i}, \mathbf{0}\right)\right) \\ \mathbf{P}_{i+1|i} &= \left(\mathbf{I} - \mathbf{K}_{i+1} \mathbf{H}_{i+1}\right) \mathbf{P}_{i+1|i} \end{aligned} \tag{3}$$

where $\mathbf{F}_i$ and $\mathbf{L}_i$ are the Jacobian matrices are evaluated at previous corrected state, while $\mathbf{H}_{i+1}$ and $\mathbf{M}_{i+1}$ are the Jacobian matrices are evaluated at current predicted state.

$$\begin{aligned} \mathbf{F}_i &= \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}_i}\right|_{\hat{\mathbf{x}}_i}, \mathbf{H}_{i+1} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{x}_{i+1}}\right|_{\hat{\mathbf{x}}_{i+1|i}}, \\ \mathbf{L}_i &= \left.\frac{\partial \mathbf{f}}{\partial \mathbf{w}_i}\right|_{\hat{\mathbf{x}}_i}, \mathbf{M}_{i+1} = \left.\frac{\partial \mathbf{h}}{\partial \mathbf{v}_{i+1}}\right|_{\hat{\mathbf{x}}_{i+1|i}} \end{aligned} \tag{4}$$

For additive noise, the Jacobian matrices $\mathbf{L}$ and $\mathbf{M}$ become identity matrices.

3. Results and Discussions

Let us consider the characteristic equation for propagation of the pure shear wave in XY plane polarized along Z,

$$\left(\frac{k}{\omega}\right)_1 = \left(\frac{\rho}{c_{44}}\right)^{1/2} \tag{5}$$

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The velocity $(V)_1$ of the pure shear wave is defined as:

$$(V)_1 = \left(\frac{\omega}{k}\right)_1 = \frac{1}{\left(\frac{\rho}{c_{44}}\right)^{1/2}} \quad (6)$$

where k is the real wave number, ω is the angular frequency, ρ is the mass density, and c_{44} is the stiffness constant of the crystal.

In this study, PZT-5H material is considered. It is assumed that the velocity of the wave propagation in the material is measured. Hence, the non-linear measurement function $\mathbf{h}(\cdot)$ is defined in Eq. (6). In Eq. (6), $\rho = 7500\ \text{kg/m}^3$ and $c_{44} = 2.3\times10^{10}\ \text{N/m}^2$ for PZT-5H material is used in the simulation of theoretical velocity[11)]. To simulate the measured velocity available in the sensor, the theoretical velocity is superimposed with noise.

For the parameter estimation using EKF, an augmented state vector $\mathbf{x}$ is defined as:

$$\mathbf{x} = [x_1] = [c_{44}] \quad (7)$$

A discrete form of the state space is given by:

$$\mathbf{x}_{i+1} = \mathbf{f}(\mathbf{x}_i, \mathbf{w}_i) = [c_{44,i}] + \mathbf{w}_i \quad (8)$$

The iteration process of the EKF is initiated by configuring $\hat{\mathbf{x}}_0 = 1\times10^{10}$ and $\mathbf{P}_0 = 10^{19}$. Further, the process noise covariance $\mathbf{Q}$ is set to be null, since the state has no superimposed noise. The measurement noise is assumed to be a Gaussian white noise process with zero mean with a standard deviation of 45. In the EKF algorithm, it is assumed that the velocity measurement is available by placing sensors at 30° angular spacing varying from 0° to 360°.

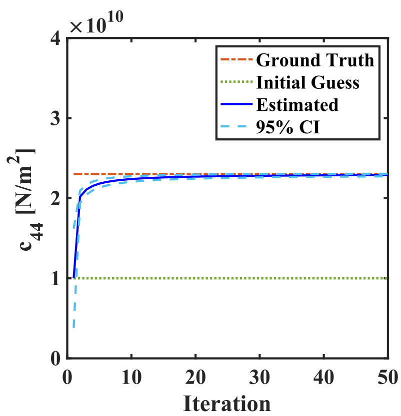

Fig. 1. Parameter (c_{44}) estimation result for the PZT-5H.

Fig. 1 illustrates the parameter estimation results in terms of the true and estimated parameter in each iteration. The dashed line in **Fig.1** depicts the 95% confidence interval (CI). This means the estimated parameter at a particular iteration lies between this interval with a probability of more than 0.95. The estimated parameter c_{44} is $2.29\times10^{10}\ \text{N/m}^2$. Therefore, the EKF estimated the parameter c_{44} with an error of about 0.43%.

4. Conclusion

In this study, the EKF is presented to estimate the mechanical properties of the PZT-5H. It has been demonstrated through the simulation studies that the measurements of velocity from the sensors are sufficient to estimate the parameter correctly. The EKF, being a Bayesian filter, can provide a confidence level for the estimate, making the algorithm robust than other deterministic approaches. However, the estimation of different parameters of the material by performing an experiment using a sparse array of sensors that needs further investigation.

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