# 3－D FDTD simulation of moving sound source and receiver 

移動音源／受音点の 3 次元 FDTD 法シミュレーション
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## 1．Introduction

Finite difference－time domain（FDTD） method［1］is a most popular numerical method for sound field analysis．In most cases in the FDTD analysis，sound source and receiver are fixed，then the response between them is mainly calculated． However，to analyze the trafic noise，or bat sonar mechanism，it is necessary to implement a moving sound sources and a receiver in FDTD method．We have implemented moving sound sources and moving sound receiving points for two－dimensional FDTD method［2］．In this paper，this scheme is extended to a three－dimensional FDTD method． Formulation and numerical experiments are carried out for the direct method［2］．The numerical accuracy is evaluated in three－dimensional case．

## 2．Theory

## 2．1 Moving sound source

To implement a moving sound source in the FDTD method，the grid points on the moving path of the sound source are driven switching every time step according to the sound source position．In the three－dimensional case，when a sound source is located between grid points，adjacent grid points are driven according to the source weighting functions． When the sound source is located at $(x, y, z)=$ $\left(x_{i}^{n}+d_{x}^{n}, y_{j}^{n}+d_{y}^{n}, z_{k}^{n}+d_{z}^{n}\right)$ as shown in Fig． 1 （a）， the position of the sound source on the local coordinate system is expressed as $\left(\xi_{i}^{n}, \eta_{j}^{n}, \varsigma_{k}^{n}\right)=$ $\left(2 d_{x}^{n} / \Delta-1,2 d_{y}^{n} / \Delta-1,2 d_{z}^{n} / \Delta-1\right)$ as shown in Fig． 1 （b），where $\Delta$ is grid interval．The source weighting fuctions are given as follows

$$
\begin{align*}
& w_{1}^{n}=\left(1-\xi_{i}^{n}\right)\left(1-\eta_{j}^{n}\right)\left(1-\varsigma_{k}^{n}\right) / 8 \\
& w_{2}^{n}=\left(1+\xi_{i}^{n}\right)\left(1-\eta_{j}^{n}\right)\left(1-\varsigma_{k}^{n}\right) / 8 \\
& w_{3}^{n}=\left(1+\xi_{i}^{n}\right)\left(1+\eta_{j}^{n}\right)\left(1-\varsigma_{k}^{n}\right) / 8 \\
& w_{4}^{n}=\left(1-\xi_{i}^{n}\right)\left(1+\eta_{j}^{n}\right)\left(1-\varsigma_{k}^{n}\right) / 8  \tag{1}\\
& w_{5}^{n}=\left(1-\xi_{i}^{n}\right)\left(1-\eta_{j}^{n}\right)\left(1+\varsigma_{k}^{n}\right) / 8 \\
& w_{6}^{n}=\left(1+\xi_{i}^{n}\right)\left(1-\eta_{j}^{n}\right)\left(1+\varsigma_{k}^{n}\right) / 8 \\
& w_{7}^{n}=\left(1+\xi_{i}^{n}\right)\left(1+\eta_{j}^{n}\right)\left(1+\varsigma_{k}^{n}\right) / 8 \\
& w_{8}^{n}=\left(1-\xi_{i}^{n}\right)\left(1+\eta_{j}^{n}\right)\left(1+\varsigma_{k}^{n}\right) / 8
\end{align*}
$$



Fig． 1 FDTD cell and local coordinates．

## 2．2 Moving receiver

In the case of a moving receiver，the basic idea is the same as that of a moving sound source．As shown in Fig． 1 （a），when the receiver is between the grid points，the sound pressure of the receiving point is interpolated from the sound pressures of the adjacent grid points using the weighting function given in Eq．（1）as follows

$$
\begin{equation*}
p^{n}=\sum_{i=1}^{8} w_{i}^{n} p_{i}^{n} \tag{2}
\end{equation*}
$$

where $p_{i}^{n}$ is sound pressure at grid point $i$ at time $n$ ．

## 3．Numerical experiments

Numerical experiments are performed by the CE－FDTD（IWB）method［3，4］．Figure 2 shows the 3－D numerical model．The grid size is $\Delta=15 \mathrm{~mm}$ ， time step is $\Delta t=44.1 \mu \mathrm{~s}$ ，and sound speed is $c_{0}=$ $340 \mathrm{~m} / \mathrm{s}$ ，so the Courant number $\chi$ is 1 ．The region is divided into $1,400 \times 1,400 \times 1,400$ FDTD cells．The boundary condition is Mur＇s first order absorbing boundary．In the figure， P is a point sound source or


Fig． 2 Numerical model for moving sound source and receiver．

> (a) moving source
> (b) moving receiver

Fig. 3 Relationship between speed of moving source or receiver and normalized Doppler shift $\left(\theta=0^{\circ}, \phi=0^{\circ}\right)$


Fig. 4 RMS error against propagation direction.
point receiver, and moves at a moving angle of $\theta$ and a constant velocity of $\boldsymbol{v} \cdot \mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are receivers when $P$ is a sound source, or are sound sources when $P$ is a receiver. The direction toward the point $Q_{1}$ is positive. The source radiates a continuous sine wave with a frequency of 500 Hz .

Figure 3 shows the result of the normalized Doppler shift for the Mach number when $\theta=$ $0^{\circ}, \phi=0^{\circ}$. Fig. (a) is the result of the moving sound source, and fig. (b) is one of the moving receiver. The solid line in the figure is the calculation result by FDTD method, and the broken line is the theoretical. The calculation results and theoretical are in good agreement with the moving sound source and receiver. It is found that both the moving source and receiver can be expressed by this method with the Doppler effect.

Figure 4 shows the root mean square (RMS) error of the calculated result of the normalized Doppler shift and the theoretical value for the azimuth angle $\phi$ when $M=0.1, \theta=0^{\circ}, 45^{\circ}$. The moving receiver has less error because the numerical dispersion error does not occur. The error is within $3 \times 10^{-3}$ in all directions. It is found that it is possible to calculate a moving source and receiver accurately.

Next, we calculated the case where the sound source and receiver move at the same time in a


Fig. 5 Moving sound source and receiver facing each other.


Fig. 6 Moving sound source and receiver on circular trajectory when there is a reflecting wall.
straight line facing each other as shown in Fig. 5 (a). The calculation conditions are the same as Fig.2. Figure (b) shows the normalized Doppler shift calculated by short-time Fourier transform of the sound pressure waveform calculated at the receiver when $M=0.1, \theta=0,45^{\circ}, \phi=0,45^{\circ}$. The RMS error is $2.00 \times 10^{-3}$ for $\theta=0^{\circ}, \phi=0^{\circ}$, and $2.23 \times 10^{-3}$ for $\theta=45^{\circ}, \phi=45^{\circ}$. The calculated result and the theoretical result again show good agreement.

Finally, we calculate a sound source-receiver pair moving in a circular orbit at a constant speed as shown in Fig. 6 (a). It is assumed that the sound source and the receiver are separated by 2 m , and its center position moves in a circular trajectory with a radius of 8 m . There is a reflective wall at a position 10.5 m from the center. Figure (b) shows the normalized Doppler shift calculated at the receiver when $M=0.1, \theta=0,45^{\circ}, \phi=0,45^{\circ}$. In both cases, the Doppler shift changes in a nearly sinusoidal shape. It is found that this method can be applied even when the sound source and receiver move in an arbitrary trajectory at the same time.

## References

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