

# Analysis for Piezoelectric Non-linear Effect on Langevin Transducer Model with Transfer Matrix Method

伝達マトリックス法を用いたランジュバン振動子における  
圧電非線形効果の影響

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## 1. Introduction

Piezoelectric material including lead zirconate titanate (PZT) is widely used in ultrasonic devices; ultrasonic machining, ultrasonic scalpel, piezoelectric buzzer and so on. These devices usually work with high power input; however, general simulation result couldn't predict actual driving performance. This difficulty comes from the nonlinear effect of piezoelectric materials, in other words, most of simulation is considered only linear effect. Non-linear effect causes significant problems such as saturated vibration velocity.

Our research group has already improved the piezoelectric LCR equivalent circuit for describing the nonlinear vibration<sup>[1]</sup>. In this circuit, the nonlinear capacitor was introduced to express the mechanical nonlinearity of piezoelectric materials. Using this model, the higher order elastic constant of piezoelectric material can be obtained by admittance curve fitting. In other report<sup>[2]</sup>, high power performance of several piezoelectric materials were compared by calculating output power and saturated vibration velocity from higher order elastic constants.

Piezoelectric device performance also should be analysed with higher elastic constant. To determine the suitable preload for Langevin transducer, the simulation model included nonlinear effect with transfer matrix method were considered in previous research<sup>[3]</sup>, however, how the material properties differences affect to the device performance was not considered.

In this study, admittance curve and vibration velocity of Langevin transducer model with various

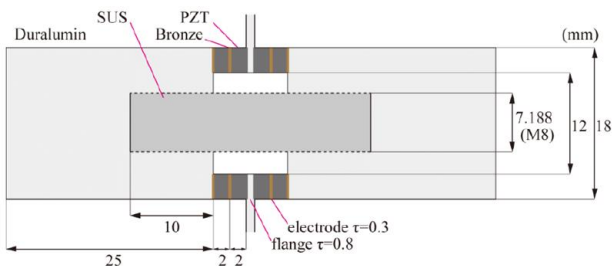


Fig. 1 Simulation model of Langevin transducer

piezoelectric materials, PZT and KNN, were calculated to clarify the relationship of material property and high power device performance.

## 2. Calculation method

### 2.1 Nonlinear model

In previous research<sup>[1]</sup>, piezoelectric equations with higher order Young's modulus were expressed as equations (1)-(2):

$$T = E_1 S + E_3 S^3, \quad E_3 = E_{3r} + jE_{3i} \quad (1)$$

$$T = E' S, \quad E' = E_1 + \frac{3}{4} E_3 |S|^2 \quad (2)$$

where,  $T$ ,  $S$ ,  $E_1$  and  $E_3$  are stress, strain, Young modulus and higher order Young modulus. In this equation, complex Young modulus,  $E'$ , was introduced as strain function. Using  $E'$ , transfer matrix for Langevin transducer can be obtained.

### 2.2 Langevin transducer model

**Fig.1** described the calculation model of Langevin transducer. In previous research<sup>[3]</sup>, optimization of preload were carried out. Transfer matrix for piezoelectric part was derived from parameters and structure. By using  $E'$  from equation (2), the nonlinear effect can be calculated.

## 3. Analysis Result

### 3.1 Piezoelectric Materials for Calculation

In this study, 3 types of Langevin transducer which has different piezoelectric materials were calculated. The piezoelectric properties for each materials are shown in **Table 1**. KNN and PZT are materials that actually exist, on the other hands, PZT NL-KNN is the virtual one whose linear parameter ( $Q_m, \epsilon_{33}^T, d_{33}$ ) are same as

Table 1 Piezoelectric properties of simulated material

Sample	$Q_m$	$d_{33}$	$\epsilon_{33}^T/\epsilon_0$	$\text{Re}(c_{11(3)}^E)$	$\text{Im}(c_{11(3)}^E)$
PZT	1300	270	1400	$-8.1 \times 10^{16}$	$6.0 \times 10^{15}$
KNN	427	123	783	$-2.7 \times 10^{17}$	$1.7 \times 10^{16}$
PZT NL- KNN	1300	270	1400	$-2.7 \times 10^{17}$	$1.7 \times 10^{16}$

PZT and nonlinear parameter (higher order elastic constant) is same as KNN. In Langevin transducer, 33 mode of piezoelectric vibration is used, so the stiffness of 33 mode should be examined, however, from the previous research<sup>[2],[4]</sup>, the order of constants of 31 mode and 33 mode are similar, therefore these calculation were carried out with 31 mode constants in this time.

### 3.2. Calculation Result

**Fig.2** and **Fig.3** show the admittance curves of PZT and KNN Langevin transducers. Similar admittance curves were observed in actual Langevin transducers in the previous report<sup>[5]</sup>. Both of these results, when the higher voltage was applied, the larger nonlinear effect was observed. Compared with voltage of 1 V<sub>pp</sub>, the admittance peak value at resonant frequency of KNN Langevin transducer is lower than that of PZT. It is because of lower  $Q_m$  and  $\varepsilon_{33}^T$  of KNN. In contrast, the admittance stability of KNN transducer with larger voltage is superior to that of PZT. Before these calculations, we expected that only the nonlinear parameters affects the resonant frequency shift and admittance peak value decreasing. On the contrary, as shown in **Fig.2 and 3**, these nonlinear effects were explicitly observed in the case of PZT type Langevin transducer even with the large absolute nonlinear piezoelectric constants. It means the overall performances of Langevin transducer is determined with total parameters combination.

**Fig.4** shows vibration velocity at maximum admittance frequency of PZT, KNN and PZT NL-KNN Langevin transducer. Comparing PZT and KNN Langevin transducers, the velocity of KNN Langevin transducer is lower than that of PZT one. On the other hands, the velocity of these two Langevin transducers get closer when the applied voltage get larger comparing with the result at lower voltage. The vibration velocity for PZT NL-KNN transducer was overtaken by KNN type at 50 V<sub>pp</sub>.

This result indicates that the saturated velocity is strongly affected by linear parameters not only by nonlinear ones. It might be explained that the nonlinear vibration comes from strains as shown in eq. 1, and the strain itself is calculated with the linear stiffness parameter. Therefore, both of linear and nonlinear stiffness parameters are dominant factors for the nonlinear piezoelectric vibration.

### 4. Conclusion

In this research, piezoelectric nonlinear properties effect to piezoelectric devices performance was studied. It revealed that the high power characteristics should be considered with not only linear and nonlinear parameter but total combination is also important.

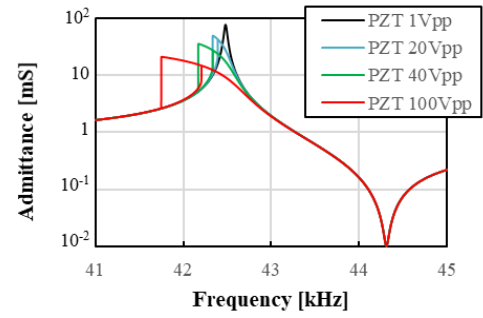


Fig. 2 Admittance curve of PZT Langevin transducer (simulation)

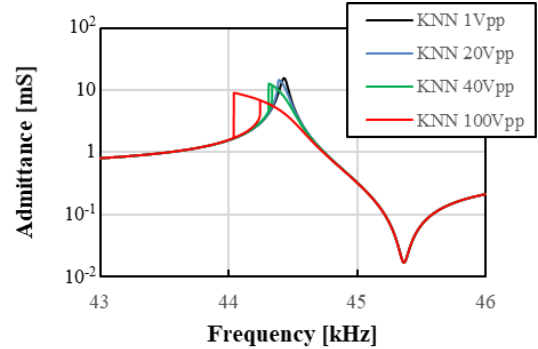


Fig. 3 Admittance curve of KNN Langevin transducer (simulation)

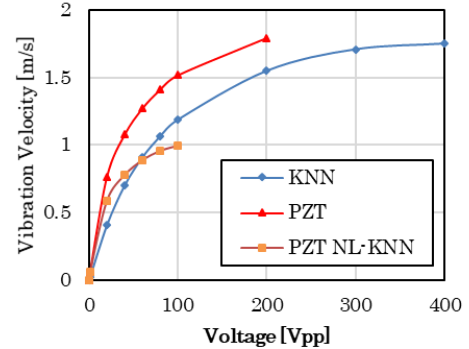


Fig. 4 Vibration velocity of PZT, KNN and PZT NL-KNN Langevin transducers (simulation)

### References

1. Y. Liu, R. Ozaki and T. Morita: *Sens. Actuat. A*, 277 (2015) 31-38.
2. S. Miyake, T. Kasashima, M. Yamazaki, Y. Okimura, H. Nagata and T. Morita, *Jpn. J. Appl. Phys.* **57** (2018) 07LB14.
3. N. Iwama, S. Miyake and T. Morita: *IEICE Tech. Rep.*, **117**(342) (2018) 37-40.
4. S. Miyake, T. Kasashima, M. Yamazaki, Y. Okimura and T. Morita: *Proceedings of JSPE Semestrial Meeting*, **2018A** (2018) 480-481.
5. N. Iwama, S. Miyake, K. Yokoyama, T. Kasashima, M. Yamazaki, Y. Okimura and T. Morita: *Proceedings of JSPE Semestrial Meeting*, **2019S** (2019) 729-730.