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Theoretical elucidation of effect of an anisotropy of shell coating ultrasound-contrast-agent on ultrasound propagation

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1. Introduction

Image resolution is drastically improved when microbubbles are used as a contrast agent in ultrasound diagnosis. The contrast bubbles are covered with a thin shell (or membrane) composed of lipids and other substances. Church [1] and Hoff et al. [2] proposed mathematical models from a mechanical point of view, assuming the shell to be a visco-elastic body (i.e., continuum), and established a pioneering theory of nonlinear oscillations of ultrasound contrast agent. However, as a critical disadvantage of previous models including Refs. [1,2], only single contrast agent is considered. The acoustic properties of multiple contrast agents are necessary because the large number of contrast agent are utilized in a clinical practice. Recently, our group proposed a mathematical model [3] that can represent the nonlinear acoustic properties of a large number of contrast agents based on mathematical model for a single contrast agent [1,2].

In general, the shell is composed of various materials such as polymers and phospholipids, which are distributed in a layered, an anisotropy thus naturally occurs and contributes to acoustic properties of bubble oscillation and ultrasound. However, all previous models (e.g., Refs. [1-3]) have assumed shell as isotropic material for simplicity. Last year, up-date equation of motion describing the oscillation of a single bubble with shell anisotropy is proposed, and the contribution of shell anisotropy to the oscillations was pointed out [6].

The purpose of this study is to extend the equation of motion for a single contrast agent incorporating shell anisotropy [6] to the case of multiple contrast agents and to clarify how shell anisotropy affects the ultrasound propagation.

2. Problem statement

The bubble is encapsulated by a visco-elastic shell, assumed to be a Kelvin-Voigt model. An anisotropy of the shell [6], which has been ignored for 25 years in previous studies, is newly incorporated. As shown in Fig. 1, the material properties are assumed to be different in the radial and orthoradial directions, which is weak assumption compared with the most general anisotropic case.

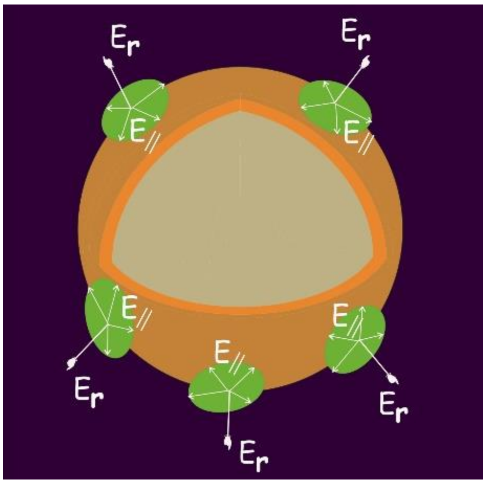

Fig. 1: Schematic illustration of anisotropy of shell encapsulating bubble [5].

We theoretically investigate the nonlinear propagation properties of ultrasound in a liquid containing a large number of bubbles encapsulated by the anisotropic visco-elastic shell. Initially, the bubbly liquid is at rest and the bubbles are uniformly distributed. The gas inside bubbles is composed of only non-condensable ideal gas, and phase change is not assumed to occur at the bubble-liquid interface. The bubbles do not coalesce, break up, appear, and extinct. Thermal conduction is dismissed for simplicity, because it is not expected to have much influence on ultrasound diagnosis.

3. Basic equations

The equation of motion for a bubble encapsulated by an anisotropic shell [6] is used:

$$\rho_{\mathrm{L0}}^{*} R^{*} \frac{D_{\mathrm{G}}^{2} R^{*}}{D t^{*2}}+\frac{3}{2} \rho_{\mathrm{L0}}^{*}\left(\frac{D_{\mathrm{G}} R^{*}}{D t^{*}}\right)^{2}$$
$$=-4 \mu_{\mathrm{L}}^{*} \frac{1}{R^{*}} \frac{D_{G} R^{*}}{D t^{*}}-p_{\mathrm{L}}^{*}-\frac{2 \sigma_{2}^{*}}{R^{*}}+p_{\mathrm{G}}^{*}-\frac{2 \sigma_{1}^{*}}{R^{*}-d_{0}^{*}}$$
$$-U^{*2} \rho_{\mathrm{L0}}^{*} K_{\mathrm{ani}}\left(1-\frac{R_{0}^{*}}{R^{*}}\right)-U^{*2} \rho_{\mathrm{L0}}^{*} C_{\mathrm{ani}} \frac{D_{G} R^{*}}{D t^{*}},$$

where t^* is the time, p^* pressure, R^* bubble radius, ρ^* density, d_0^* initial shell thickness, U^* typical propagation speed of the wave, μ^* viscosity, σ_1^* and σ_2^* are surface tensions at the internal and external boundaries of the shell, respectively; the subscripts G and L denote volume-averaged variables in gas and liquid phases, respectively, the subscript 0 denotes the quantities in the initial uniform state at rest, and the superscript * denotes a dimensional quantity. Here, anisotropic dissipation factor C_{ani} and anisotropic elastic constant K_{ani} are constants determined from the viscosity and the elastic constants of the shell, respectively; K_{ani} is given by

$$K_{\mathrm{ani}} = \left\{ \begin{array}{l} {R_{20}^*}^{\beta_+}\left[(1-\nu_{||}^*)E_r^*\beta_- + 2\nu_{\theta r}^* E_{||}^*\right]\left({R_{20}^*}^{\beta_- -1} - {R_{10}^*}^{\beta_- -1}\right) \\ + {R_{20}^*}^{\beta_-}\left[(1-\nu_{||}^*)E_r^*\beta_+ + 2\nu_{\theta r}^* E_{||}^*\right]\left({R_{10}^*}^{\beta_+ -1} - {R_{20}^*}^{\beta_+ -1}\right) \end{array} \right\} \times \frac{1}{\rho_{L0}^* {U^*}^2\left(1-\nu_{||}^* - 2E_{||}^*{\nu_{\theta r}^*}^2/E_r^*\right)\left({R_{10}^*}^{\beta_-}{R_{20}^*}^{\beta_+} - {R_{10}^*}^{\beta_+}{R_{20}^*}^{\beta_-}\right)},$$

where E_r^* is the Young modulus in the radial direction, $\nu_{\theta r}^*$ the Poisson ratio with radial load, and $E_{||}^*$ and $\nu_{||}^*$ are the Young modulus and the Poisson ratio in the orthoradial plane, respectively.

Furthermore, to close the set of equations, the conservation equations of mass and momentum for bubbly liquid based on two fluid-model [3,4] are used:

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) = 0,$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] = 0,$$

$$\frac{\partial}{\partial t^*}(\alpha\rho_{\mathrm{G}}^* u_{\mathrm{G}}^*) + \frac{\partial}{\partial x^*}\left(\alpha\rho_{\mathrm{G}}^* {u_{\mathrm{G}}^*}^2\right) + \alpha\frac{\partial p_{\mathrm{G}}^*}{\partial x^*} = F^*,$$

$$\frac{\partial}{\partial t^*}[(1-\alpha)\rho_{\mathrm{L}}^* u_{\mathrm{L}}^*] + \frac{\partial}{\partial x^*}\left[(1-\alpha)\rho_{\mathrm{L}}^* {u_{\mathrm{L}}^*}^2\right] + (1-\alpha)\frac{\partial p_{\mathrm{L}}^*}{\partial x^*} + P^*\frac{\partial \alpha}{\partial x^*} = -F^*,$$

where α is the volume fraction of gas phase, x^* space coordinate, u fluid velocity, P^* surface-averaged pressure, and F^* interfacial momentum transport [3,4].

4. Results

We successfully derived the KdV-Burgers equation including the effect of shell anisotropy in terms of the variation of bubble radius R_1:

$$\frac{\partial R_1}{\partial \tau} + \Pi_1 R_1 \frac{\partial R_1}{\partial \xi} + \Pi_2 \frac{\partial^2 R_1}{\partial \xi^2} + \Pi_3 \frac{\partial^3 R_1}{\partial \xi^3} = 0,$$

$$\tau \equiv \epsilon t, \qquad \xi \equiv x - (1+\epsilon\Pi_0)t,$$

where τ and ζ are independent variables through the variable transformation, and ϵ is the nondimensional wave amplitude. Here, coefficient of dissipation term is given by

$$\Pi_2 = -\frac{1}{6\alpha_0}\left(\frac{{U^*}^2 {R_0^*}^3 {\omega_{\mathrm{B}}^*}^2}{c_{\mathrm{L0}}^* \epsilon {L^*}^4} + \frac{4\mu_{\mathrm{L}}^*}{{U^*}^2 \rho_{\mathrm{L0}}^* T^*} + C_{\mathrm{ani}} - \frac{8\sigma_1^*}{(R_0^* - d_0^*){U^*}^2 \rho_{\mathrm{L0}}^*}\right),$$

where L^* is a typical wavelength, c_{L0}^* the speed of sound in the liquid, ω_{B}^* an eigenfrequency of the encapsulated bubble, and T^* a typical period of the wave concerned. Further, Π_2 contains C_{ani} and is affected by the elastic modulus of the shell anisotropy, and K_{ani} affects Π_1. Since explicit forms of C_{ani} and nonlinear term coefficient Π_1 are quite complex, it is not shown here. The explicit form of advection term coefficient Π_0 and dispersion term coefficient Π_3 are the same as counterparts in Ref. [4], but are affected by K_{ani}.

5. Summary

The equation of motion for a single bubble encapsulated by a visco-elastic shell with an anisotropy [6] was extended to the case of multiple bubbles, and we have derived the equation for a nonlinear propagation of ultrasound in liquid containing multiple encapsulated bubbles. As a result, shell anisotropy contributed to advection, nonlinear dispersion, and dissipation effects; especially, large contribution of dissipation was predicted. A quantitative discussion will be presented in a presentation.

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