

Numerical Simulation of Piezoelectric Signals Generated in Cancellous Bone by Ultrasound Irradiation: Effect of Microstructure

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1. Introduction

On the fact that the bone formation can be driven by mechanical loads,¹ the medical healing of bone fracture using low-intensity pulsed ultrasound (LIPUS) has been realized.^{2,3} The bone formation can be accompanied by the piezoelectric effects.⁴ In the author's study, the piezoelectric properties in bone, mainly in cancellous bone, at ultrasound frequencies has been investigated using piezoelectric finite-difference time-domain (PE-FDTD) simulations.⁵

As the piezoelectric signal in cancellous bone is weak and sensitive to the trabecular structure, it is difficult to clarify the effect of the microstructure. In this study, using numerical models of water-saturated cancellous bone, in which the trabecular elements were regularly eroded by an image processing technique,⁶ the effects of the microstructure were investigated.

2. Methods

The governing equations of the PE-FDTD method are given as⁵

$$\rho \frac{\partial \dot{u}_i}{\partial t} = \frac{\partial \tau_{ii}}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \tau_{ik}}{\partial x_k}, \quad (1)$$

$$\frac{\partial \tau_{ii}}{\partial t} = (\lambda + 2\mu) \frac{\partial \dot{u}_i}{\partial x_i} + \lambda \frac{\partial \dot{u}_j}{\partial x_j} + \lambda \frac{\partial \dot{u}_k}{\partial x_k} - e_{ii} \frac{\partial E_i}{\partial t} - e_{ji} \frac{\partial E_j}{\partial t} - e_{ki} \frac{\partial E_k}{\partial t}, \quad (2)$$

$$\frac{\partial \tau_{jk}}{\partial t} = \mu \left(\frac{\partial \dot{u}_j}{\partial x_k} + \frac{\partial \dot{u}_k}{\partial x_j} \right) - e_{il} \frac{\partial E_l}{\partial t} - e_{jl} \frac{\partial E_j}{\partial t} - e_{kl} \frac{\partial E_k}{\partial t}, \quad (3)$$

$$\begin{aligned} \varepsilon_{ii} \frac{\partial E_i}{\partial t} = & -e_{ii} \frac{\partial \dot{u}_i}{\partial x_i} - e_{ij} \frac{\partial \dot{u}_j}{\partial x_j} - e_{ik} \frac{\partial \dot{u}_k}{\partial x_k} \\ & - \frac{e_{il}}{2} \left(\frac{\partial \dot{u}_j}{\partial x_k} + \frac{\partial \dot{u}_k}{\partial x_j} \right) - \frac{e_{im}}{2} \left(\frac{\partial \dot{u}_k}{\partial x_i} + \frac{\partial \dot{u}_i}{\partial x_k} \right), \quad (4) \\ & - \frac{e_{in}}{2} \left(\frac{\partial \dot{u}_i}{\partial x_j} + \frac{\partial \dot{u}_j}{\partial x_i} \right) + \frac{\partial D_i}{\partial t} \end{aligned}$$

$$\frac{\partial D_i}{\partial t} = -\sigma_i E_i. \quad (5)$$

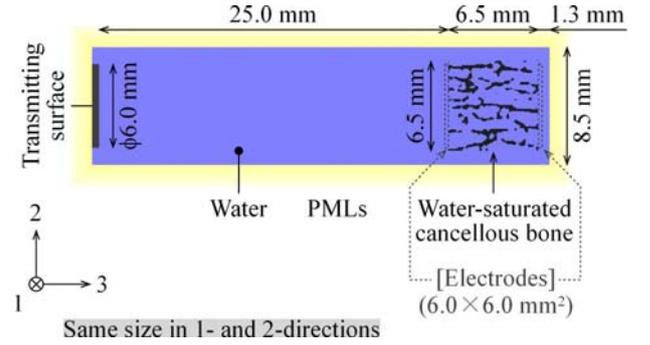


Fig. 1 Numerical model for simulating piezoelectric signals generated in cancellous bone by ultrasound irradiation.

Here, \dot{u}_i is the particle velocity, τ_{ii} and τ_{ij} are the normal and shear stresses, respectively, E_i is the electric field, and D_i is the electric displacement. ρ is the density, λ and μ are the first and second Lamé coefficients, respectively, e_{ij} is the piezoelectric constant, ε_{ii} is the dielectric constant, and σ_i is the conductivity.

A cubic cancellous bone model with a side of about 6.5 mm was reconstructed from the X-ray microcomputed tomographic (μ CT) image of bovine bone. It was assumed that the pore spaces were saturated with water. The porosity was about 0.76 (76%). Using the image processing technique in the previous study,⁶ the trabecular elements in the cancellous bone model were eroded. Three erosion procedures A, B, and C were used to realize distinct changes in the microstructure with increasing porosity. In the procedure A, the erosions applied to the points of the solid bone, that is the transformations of the surface points of the trabecular elements from the solid bone to the pore space (water), were randomly distributed in every direction. In the procedure B, the distribution of the erosions was weighted in the direction parallel to the major trabecular orientation. In the procedure C, the distribution was weighted in the perpendicular direction.

Figure 1 shows the numerical model for the PE-FDTD simulation using the cancellous bone models. The ultrasound wave was irradiated in the direction of the major trabecular orientation. The experimental data of the burst wave at 1 MHz, which corresponded to the irradiated ultrasound

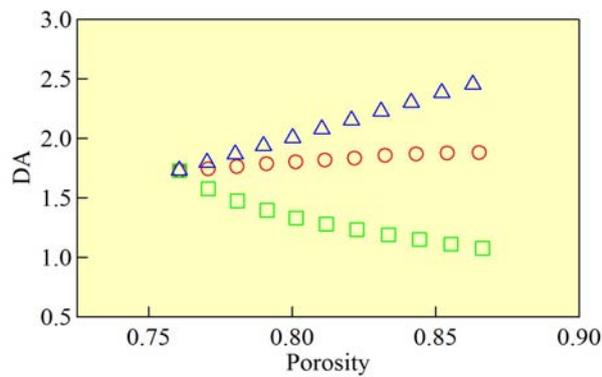


Fig. 2 Variations in degree of anisotropy (DA) of the cancellous bone models with porosity by erosion procedures A (circles), B (squares), and C (triangles).

signal, was applied to the normal stress components τ_{ii} on the transmitting surface. The piezoelectric signal was calculated from the electric fields E_i in the trabecular elements between the electrodes. Then, the electrodes were regarded as perfect conductors, and the elastic properties were ignored.

3. Results and Discussion

To quantitatively estimate the change of the microstructure in the cancellous bone model by each erosion procedure, the degree of anisotropy (DA), that is the ratio of the maximum to the minimum mean intercept lengths (MILs)⁷ of the trabecular elements, was measured. Figure 2 shows the DA as a function of porosity. The circular, square, and triangular marks show the DA in the procedures A, B, and C, respectively, which is a common notation in subsequent figures. In the procedures A, B, and C, the DA scarcely varied, decreased, and increased with porosity, respectively.

Figures 3 and 4 show the amplitudes of the simulated piezoelectric signals as functions of porosity and DA, respectively. In all procedures, the piezoelectric signal amplitudes were little correlated with both porosity and DA. However, it can be shown in Fig. 4 that the amplitude variation was small in the procedure A in which the DA variation was small, but was large in the procedures B and C in which the DA variation was large. Therefore, it was considered that the random variation of the piezoelectric signal amplitude was due to some change of the microstructure caused by the erosion of the trabecular elements.

4. Conclusions

In the simulated results obtained using the PE-FDTD method, it was suggested that the piezoelectric signal amplitude in water-saturated

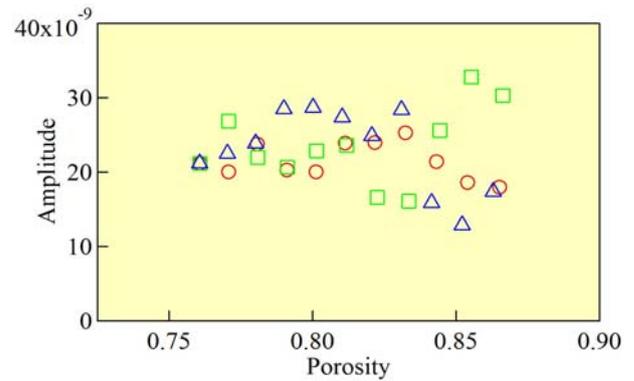


Fig. 3 Variations in amplitudes of simulated piezoelectric signals with porosity by erosion procedures A (circles), B (squares), and C (triangles).

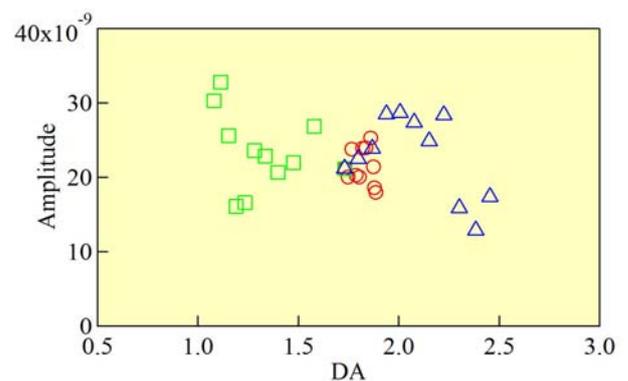


Fig. 4 Variations in amplitudes of simulated piezoelectric signals with degree of anisotropy (DA) by erosion procedures A (circles), B (squares), and C (triangles).

cancellous bone could affect the microstructure. The causal structural parameter should be identified, which will be the subject in a future study.

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