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Examination of statistical limitation in statistics evaluation of ultrasound echo envelope amplitudes

Shohei Mori[1†], Mototaka Arakawa[2,1], Hiroshi Kanai[1,2], and Hiroyuki Hachiya[3] ([1]Grad. Sch. Eng. Tohoku Univ.; [2]Grad. Sch. Biomed. Eng. Tohoku Univ.; [3]Grad. Sch. Eng. Tokyo Tech)

1. Introduction

A statistics evaluation for ultrasound echo envelope amplitudes contributes to the quantitative tissue characterization for diseases such as liver fibrosis [1]. In the statistics-based evaluation, there exists a statistical limitation due to the finite data length. In the present study, we formulated the statistical variance of the moment depending on the data length. Moreover, we examined the relationship between the analysis conditions for ultrasound echo envelopes and the statistical variances of the moment.

2. Principles

2.1 Rayleigh distribution

The statistics of echo envelopes reflect the characteristics of scatterer distribution. For example, when the scatterers with sufficiently smaller than the area of point spread function (PSF), A_{PSF}, are randomly, densely (more than 10 scatterers/A_{PSF}), and homogeneously distributed, the obtained echo envelope amplitude x_{RA} follows a Rayleigh distribution [1],

$$p_{\mathrm{RA}}(x_{\mathrm{RA}}) = \frac{2x_{\mathrm{RA}}}{\sigma_{\mathrm{RA}}} \exp\left(-\frac{x_{\mathrm{RA}}^2}{\sigma_{\mathrm{RA}}^2}\right), \qquad (1)$$

where σ_{RA} is a scale parameter.

2.2 Statistical moment

The moment is an indicator for evaluating the statistical property of echo envelopes. The theoretical k th-order of moment $M_{\mathrm{T}}(k;p)$ of random variables following the probability distribution $p(x)$ is defined by

$$M_{\mathrm{T}}(k;p) = \mathrm{E}[x^k] = \int_{-\infty}^{\infty} x^k \cdot p(x)dx\,, \qquad (2)$$

where x is the variable following $p(x)$ and $\mathrm{E}[\cdot]$ is an operation of expectation. When x_{RA} follows the Rayleigh distribution $p_{\mathrm{RA}}(x_{\mathrm{RA}})$ in Eq. (1), the theoretical moment is given by [1]

$$M_{\mathrm{T}}(k;p_{\mathrm{RA}}) = \int_{-\infty}^{\infty} x_{\mathrm{RA}}^k \cdot p_{\mathrm{RA}}(x_{\mathrm{RA}})dx_{\mathrm{RA}}$$
$$= \Gamma\left(1+\frac{k}{2}\right) \cdot \sigma_{\mathrm{RA}}^k, \qquad (3)$$

where $\Gamma(\cdot)$ is a gamma function.

2.3 Statistical variance of moment

The theoretical moment $M_{\mathrm{T}}(k;p)$ in Eq. (2) is given under the ideal condition of the unlimited data length; however, in actual cases, the data length is finite. Thus, let us define the kth-order of moment calculated from the L random variables following $p(x)$ as

$$M_L(k;p) = \mathrm{E}_{l\in L}\left[x_l^k\right], \qquad (4)$$

where x_l is the l-th sample following $p(x)$. Let us describe x_l^k by

$$y_{k,l} = x_l^k. \qquad (5)$$

Since x_l is generated from the single population following $p(x)$, $y_{k,l}$ is also generated from the single population following $p_k(y_k)$. Therefore, from the central limit theorem, the average of $y_{k,l}$ for L independent samples, $\mathrm{E}_{l\in L}\left[y_{k,l}\right]$, follows the normal distribution $N\left(\mu_{y_k}, \sigma_{y_k}^2/L\right)$ as

$$\mathrm{E}_{l\in L}\left[y_{k,l}\right]$$
$$= \mathrm{E}_{l\in L}\left[x_l^k\right] = M_L(k;p) \in N\left(\mu_{y_k}, \frac{\sigma_{y_k}^2}{L}\right), \qquad (6)$$

$$N\left(\mu_{y_k}, \frac{\sigma_{y_k}^2}{L}\right) = \frac{1}{\sqrt{2\pi\sigma_{y_k}^2}} \exp\left(-\frac{\left(x-\mu_{y_k}\right)^2}{2\sigma_{y_k}^2}\right). \qquad (7)$$

Here, μ_{y_k} and $\sigma_{y_k}^2/L$ are the average and variance of the normal distribution, respectively, and given by

$$\mu_{y_k} = \mathrm{E}[y_k] = \mathrm{E}[x^k] = M_{\mathrm{T}}(k;p), \qquad (8)$$

$$\sigma_{y_k}^2 = \mathrm{E}\left[\left(y_k-\mu_{y_k}\right)^2\right] = \mathrm{E}\left[\left(x^k-\mathrm{E}[x^k]\right)^2\right]$$
$$= M_{\mathrm{T}}(2k;p) - M_{\mathrm{T}}(k;p)^2. \qquad (9)$$

Thus, the kth order of the moment $M_L(k;p)$ calculated from the L independent samples following $p(x)$ follows the normal distribution,

$$M_L(k;p)$$
$$\in N(M_{\mathrm{T}}(k;p), \{M_{\mathrm{T}}(2k;p) - M_{\mathrm{T}}(k;p)^2\}/L). \qquad (10)$$

This relationship shows that the moment $M_L(k;p)$ calculated from the L independent samples statistically fluctuates with the variance of $\{M_{\mathrm{T}}(2k;p) - M_{\mathrm{T}}(k;p)^2\}/L$, even if all L samples follow $p(x)$ and completely independent of each other. This is the statistical limitation in the statistical evaluation for the ultrasound echo envelopes.

3. Methods

3.1 Effective data length in the analyzed region for ultrasound echo envelopes

In the evaluation for the ultrasound echo envelopes, the sampling intervals in the depth and

mori@tohoku.ac.jp

lateral directions are generally narrower than the pulse length and beam width, respectively; therefore, the samples in the region of interest (ROI) have a dependence on neighboring samples. Therefore, to determine the statistical variance of the moment by Eq. (10), we have to know the effective data length in the ROI, L_{ROI}, which is regarded as independent of each other.

This effective data length in the ROI is determined by the relationship between the ROI size and the shape of PSF [2]. Thus, we define the effective data length in the ROI, L_{ROI}, as a ratio of the ROI area A_{ROI} to the PSF area A_{PSF} as

$$L_{\mathrm{ROI}}(\rho) = \frac{A_{\mathrm{ROI}}}{A_{\mathrm{PSF}}(\rho)}, \tag{11}$$

$$A_{\mathrm{PSF}}(\rho) = Area\left(\mathrm{PSF} > \max(\mathrm{PSF}) \cdot 10^{-\frac{\rho}{20}}\right), \tag{12}$$

where $A_{\mathrm{PSF}}(\rho)$ is the area of PSF larger than $-\rho$ dB of maximum amplitude of the PSF.

3.2 Simulation conditions

To examine the relationship between the effective data length $L_{\mathrm{ROI}}(\rho)$ and the statistical variances of the moment, $M_L(k; p_{\mathrm{RA}})$, we simulated the ultrasound radiofrequency (RF) signals received from the randomly and densely distributed scatterer distribution, using Field II simulation tool [3,4]. The scatterers were distributed to be that the obtained echo envelopes follow the Rayleigh distribution $p_{\mathrm{RA}}(x)$ in Eq. (1). The transmitted and sampling frequencies were set to 7.5 and 40 MHz, respectively, and the beam spacing was set to 0.3 mm.

Figure 1 shows an example of the simulated B-mode image. In Fig. 1, we set several widths of ROI and calculated the 1st and 3rd moments by Eq. (4). To exclude the effect of change of acoustic field in the depth direction, only the width of ROI was changed from 1.2 to 48.5 mm while the height of ROI was fixed at 3 mm. The ultrasound RF signals were simulated 500 times by randomly changing the scatterer distribution.

4. Results and Discussion

Figure 2 shows the variance of moments. The results for the simulated echo envelopes are shown by dotted markers and the theoretical variance calculated by substituting Eq. (3) into Eq. (10) is plotted with the black straight line. The horizontal axes for the simulation results are shown by the data length in the ROI (blue), and the effective data length in the ROI calculated by $L_{\mathrm{ROI}}(\rho = 3)$ (green) or $L_{\mathrm{ROI}}(\rho = 6)$ (red) using Eq. (11).

As shown in Fig. 2, the variance of moments of simulated echo envelopes decreased as the ROI width became wider. When the data length in the ROI is regarded as the length of independent data, L, the statistical variance of moments (blue) largely deviated from the theoretical value (black straight line). Comparing the results for $\rho = 3$ (green) and

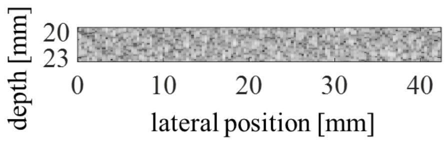

Fig. 1. Simulated ultrasound B-mode image.

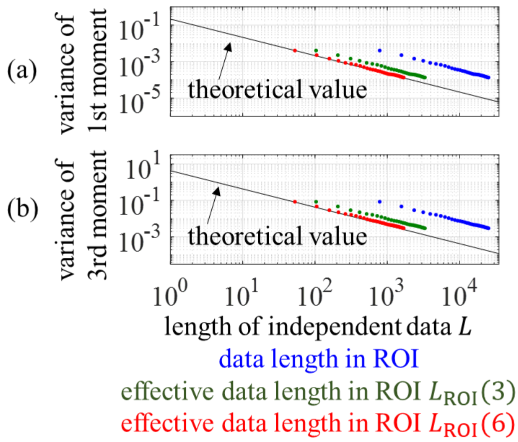

Fig. 2. Relationship between data length and variances of (a) 1st and (b) 3rd moments. Straight line: theoretical value, dotted marker: simulation results with different horizontal axes.

$\rho = 6$ (red) in Eq. (11), the latter corresponded to the theoretical value. Thus, the ratio of the ROI area A_{ROI} to the PSF area $A_{\mathrm{PSF}}(\rho = 6)$ was related to the length of independent data, L.

Thus, the statistical variance of the moment is related to the relationship between the ROI size and the PSF. If the statistical variance is too large for evaluating the tissue characteristics, the ROI size should be broadened, or the area of PSF should be narrowed (*i.e.*, the ultrasound spatial resolution should be improved).

4. Conclusion

In the present study, the statistical variance of the moment was formulated and compared with the analyzed conditions for ultrasound echo envelopes. The effect of this statistical variance on tissue characterization will be evaluated in our future study.

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