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Resonance and absorption of ultrasonic waves in asymmetric viscoelastic-elastic laminates

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1. Introduction

Multilayer structures can exhibit unique acoustic characteristics depending on the properties and sequence of the layers. A wide variety of studies[1] has been carried out so far to facilitate the understanding of acoustic wave propagation in layered structures, which helps to design various acoustic devices.

However, wave propagation in asymmetric layered structures remains as a phenomenon to be resolved. When a laminate consists of elastic and viscoelastic layers, the effect of the viscoelasticity on the wave propagation is not sufficiently clear yet. Lavrentyev and Rokhlin[2] investigated ultrasonic responses of a viscoelastic layer between two semi-infinite dissimilar materials, showing that the damping in the layer affects the wave resonance characteristics. Recently, Mori *et al.*[3] examined wave propagation in metal-plastic laminates immersed in water and showed that the viscoelasticity of the plastic layer contributes to the appearance of resonance in the reflection spectra.

In this paper, ultrasonic resonance and absorption in asymmetric viscoelastic-elastic laminates is investigated by extending the theoretical formulation in Ref. 3. The viscoelastic effect emerging in the ultrasonic resonance and absorption is examined in a theoretical manner.

2. Theoretical Model and Formulation

As shown in **Fig. 1**, a bilayer laminate embedded in an infinite homogeneous medium is considered in this study. The laminate consists of linear viscoelastic and purely elastic layers with thicknesses d_V and d_E, respectively. The layers are assumed to be planar and homogeneous. The x axis corresponds to the thickness direction of the laminate.

When a longitudinal plane wave propagating in the x direction is incident on the laminate, the wave propagation is substantially one-dimensional. In the frequency domain, the stress-strain relation of the viscoelastic layer is given by

$$\sigma = (E - \mathrm{i}D)\varepsilon, \tag{1}$$

when the time-dependent terms are represented by $\exp(-\mathrm{i}\omega t)$, where $\mathrm{i} = \sqrt{-1}$, ω is angular frequency, and t is time. The quantities σ and ε are normal stress and normal strain, respectively, and $E - \mathrm{i}D$ (E, $D > 0$) is complex elastic modulus. The wavenumber in the viscoelastic layer is expressed as

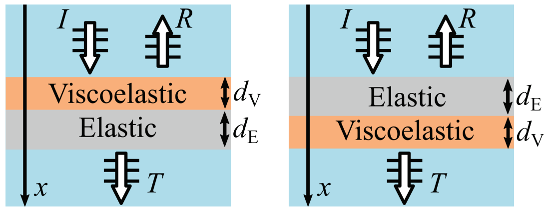

Fig. 1 Asymmetric bilayer laminate subjected to wave incidence in an infinite medium.

$$k = \frac{\omega}{c\sqrt{1 - \mathrm{i}\zeta}}, \tag{2}$$

where $c = (E/\rho)^{1/2}$ is wave velocity, ρ is mass density, and $\zeta = D/E$ is loss factor. In this paper, the frequency dependence of the complex elastic modulus is not considered. Namely, the coefficients E and D do not depend on ω.

Based on the formulation in Refs. 1 and 3, the reflection spectrum is expressed by

$$R = \frac{z_{321}^{\mathrm{in}}(\omega) - z_1}{z_{321}^{\mathrm{in}}(\omega) + z_1}, \tag{3}$$

where z_1 is the acoustic impedance of the infinite medium, and $z_{321}^{\mathrm{in}}(\omega)$ is the effective acoustic impedance of the layers, which depends on the angular frequency ω. The specific form of $z_{321}^{\mathrm{in}}(\omega)$ can be found in Refs. 1 and 3.

3. Results

3.1 *Model validation by experimental results*

To validate the theoretical model, the reflection spectra measured for a polystyrene (PS)-aluminum alloy (AL) bonded laminate[3] are compared to the theoretical results obtained by Eq. (3). The PS and AL layers are modeled as viscoelastic and purely elastic layers with thicknesses d_V = 1.2 [mm] and d_E = 2.0 [mm], respectively. The wave velocities of AL and PS and the loss factor of PS were set as c_E = 6.41 [km/s], c_V = 2.16 [km/s], and ζ = 0.02 based on the measured results for single plates of AL and PS, respectively.

Fig. 2 (a) and (b) show the amplitudes of the reflection spectra $|R|$ for wave incidence from the sides of the AL and PS layers, respectively. These figures demonstrate that the theoretical curves obtained by Eq. (3) well reproduce the frequency dependences of the experimental data. In Fig. 2(a), the reflection spectrum for the wave incidence from

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the AL layer takes local minima at the resonance frequencies of the AL layer $f_{En} = nc_E/2d_E$, where n is a positive integer. For the wave incidence from the PS layer side, on the other hand, local minima appear at the resonance frequencies of the PS layer $f_{Vn} = (2n - 1)c_V/4d_V$ as well as f_{En} in Fig. 2(b). The damping effect of the PS layer results in the appearance of local minima in the reflection spectrum.[3]

3.2 *Effect of loss factor on resonance and absorption*

Based on the theoretical formulation in Sec. 2, the effect of the loss factor ζ on the reflection spectrum $|R|$ is theoretically examined for the wave incidence from the viscoelastic layer. In this section, the thickness of the viscoelastic layer is set as $d_V = d_E c_V/c_E = 0.674$ [mm], which represents that at the resonance frequency of the viscoelastic layer f_{Vn}, the elastic layer shows anti-resonance.

Fig. 3 shows the amplitudes of the reflection spectrum $|R|$ calculated by Eq. (3) for different loss factors ζ. The result for $\zeta = 0$ corresponds to a purely elastic laminate, showing the local minima only at the resonance frequencies f_{En}. As the loss factor ζ increases, local minima appear at the resonance frequencies of the viscoelastic layer f_{Vn}. However, the depth of the trough at the resonance frequencies f_{Vn} does not vary monotonically with the increase of the loss factor ζ. The trough depth at the first-order resonance frequency f_{V1} nominally decreases with increasing loss factor, but the change at the fourth order resonance frequency f_{V4} is not monotonic.

The local minima of the reflection spectra at different resonance frequencies of the viscoelastic layer, denoted as $|R_V|$, are calculated for different loss factors ζ, as shown in **Fig. 4**. It is found in this figure

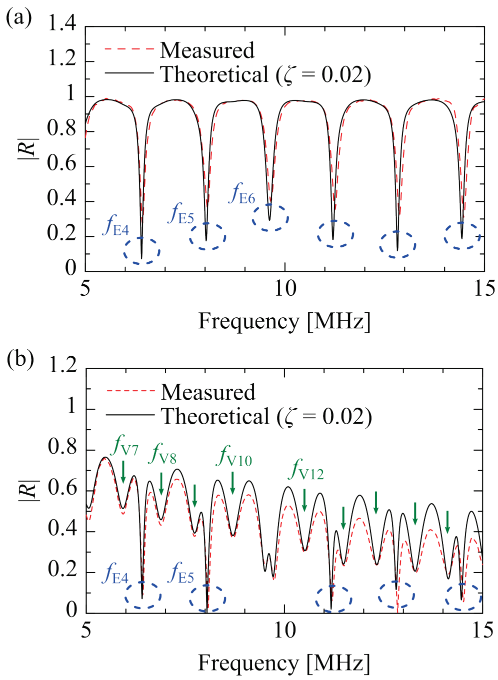

Fig. 2 Reflection spectra of a PS-AL laminate for wave incidence from the (a) AL and (b) PS layers.

that $|R_V|$ for each order resonance frequency f_{Vn} has a minimum at a certain loss factor. In particular, the local minimum reflection spectrum $|R_V|$ for the second to fourth-order resonance frequencies (n = 2, 3, and 4) shows a distinct drop of less than –50 dB. This behavior is probably associated with critical attenuation,[2] which results from the combination of the destructive interference in the reflected wave components and the wave damping in the viscoelastic layer.

4. Summary

In this paper, the resonance and absorption behavior in viscoelastic-elastic bilayer laminates has been examined theoretically. The effect of the loss factor on the amplitude of the reflection spectrum has been investigated for wave incidence from the viscoelastic layer side. It has been shown that when the two layers satisfy a specific condition, the resonance of the viscoelastic layer leads to almost zero reflection at a certain loss factor.

References

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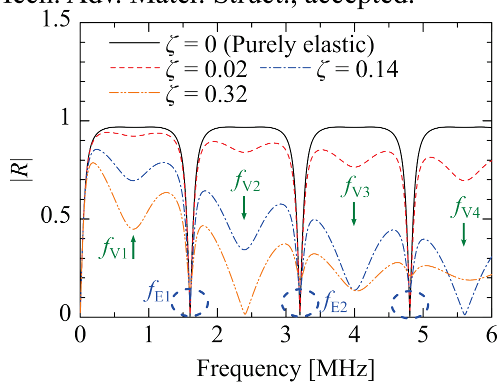

Fig. 3 Amplitudes of the theoretical reflection spectra for wave incidence from the viscoelastic layer, obtained for different loss factors.

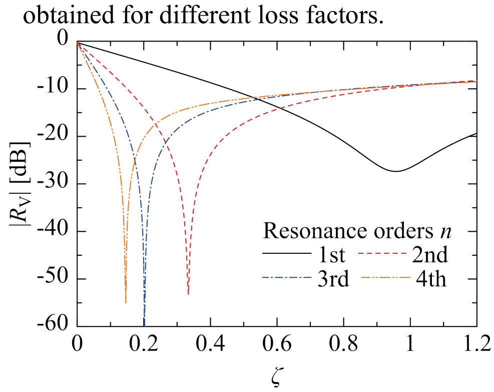

Fig. 4 Variation of the local minimum reflection spectra with the loss factor for different order resonance frequencies of the viscoelastic layer.