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Resonance profiles and complex Fano parameters in a weakly coupled oscillator system with degenerate eigenfrequencies

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1. Introduction

The Fano resonance [1, 2] originates from the constructive and destructive interference of a discrete localized state with a continuum of propagating modes that share the same frequency. That is, Fano resonance is a general wave phenomenon that does not depend on the nature of the waves. Thus, this resonance has also been reported in various classical systems such as photonic crystals and phononic metamaterials [2, 3, 4] in addition to various artificial quantum structures. In addition, Fano resonances have a wide range of applications, such as sensing and switching. In particular, the manipulation of the line shape of Fano resonances has useful applications with significant flexibility.

Suppression of the systems response due to destructive interference in Fano resonance has been explained using a simple model consisting of two weakly coupled harmonic oscillators [5]. Even in such a simple system, however, the expression for the resonance profile was not derived. Also, Fano parameters have been implicitly treated as real numbers in many previous studies. In the previous work [6], we analytically examined the resonance generated in two weakly coupled harmonic oscillators. We derived the analytical expressions for the amplitude profiles near the resonant frequencies and showed that the resonance can be generally described by a Fano formula with a complex Fano parameter. However, our formula can be used for the oscillators systems consisting of non-equivalent oscillators.

In the present paper, we consider a system in which a periodic external force is applied to weakly coupled two equivalent oscillators whose eigenfrequencies are degenerate.

2. Model and theory

In this study, we examine the dynamics of a pair of equivalent harmonic oscillators connected by a weak spring. The parameters that specify our system are as follows: coupling constant $V(\neq 0)$; eigenfrequency ω_i of the oscillator $i\ (=1,2)$ when there is no friction; the friction coefficient γ_i. Also, a periodic external force with a frequency ω is assumed to act on the oscillator 1. The stationary solutions can be expressed in the form of $x_i = c_i e^{i\omega t}$.

The analytical expressions for the complex amplitude c_1 and c_2 near the resonant frequency can be expressed in the form of the Fano and Lorentz formulae, respectively:

$$c_1 = \varsigma(\omega_i)\frac{\omega-\omega_i^R+q_i\Gamma_i}{\omega-\omega_i^R-i\Gamma_i}, \tag{1}$$

$$c_2 = \sigma(\omega_i)\frac{1}{\omega-\omega_i^R-i\Gamma_i}. \tag{2}$$

The explicit expressions for the resonance frequency ω_i^R, resonance width Γ_i, Fano parameter q_i, and ς and σ are given in Ref. [6]. These expressions can be applicable for the case where the eigenfrequencies of the two oscillators are not degenerate. When they are degenerate, different calculation methods are required.

We first used perturbation theory to determine the eigen-frequencies split by the interaction V for two non-friction equivalent oscillators, and then used those frequencies to calculate the resonance profile. For this case, the Fano parameter is also generally represented by a complex number.

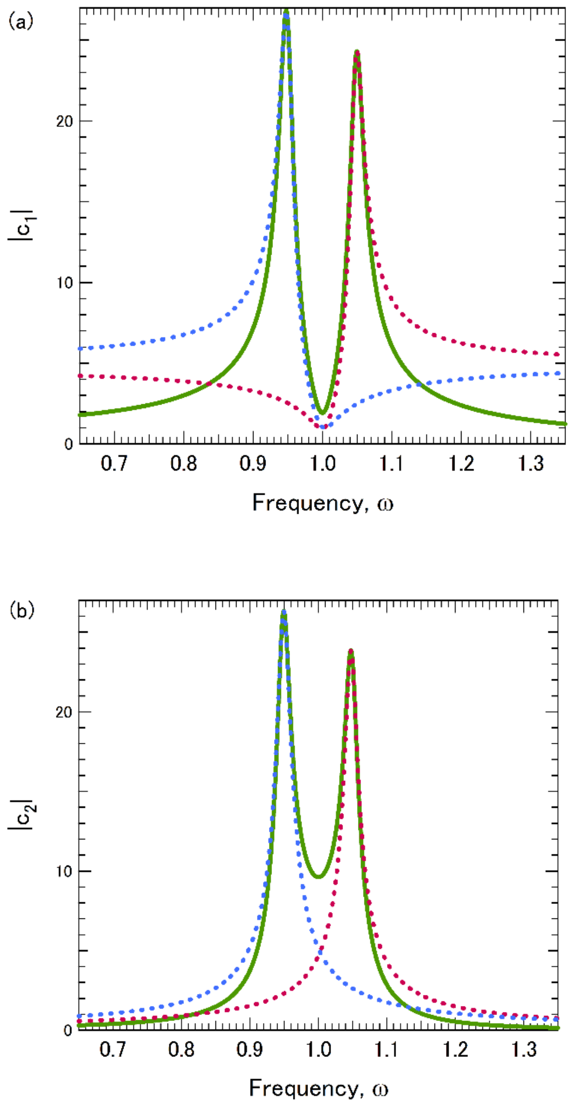

Fig. 1 The amplitudes of the first oscillator (a) and second one (b), which are calculated for $\gamma_1 = \gamma_2 = 0.01$, $\omega_1 = \omega_2 = 1.0$, and $V = 0.1$. The solid lines represent the numerically calculated values, whereas the dotted lines represent the analytically calculated values.

3. Results and discussions

Figure 1 (solid line) shows the amplitudes calculated for $\gamma_1 = \gamma_2 = 0.01$, $\omega_1 = \omega_2 = 1.0$ and $V = 0.1$. The degenerated eigenfrequency ω_1 is split into $\omega_+ = \omega_1 + V/(2\omega_1) = 1.10$ and $\omega_- = \omega_1 - V/(2\omega_1) = 0.90$ by interaction V. For the oscillator 1, the amplitude $|c_1|$ has asymmetric profiles around $\omega_\pm$ but does not become zero. These asymmetric profiles can be described by a Fano formula with a complex Fano parameter. We can calculate the resonance frequencies and widths and Fano parameters around $\omega_\pm$ by using our formulae. The calculated values are as follows: $\Gamma_- = 0.01$, $\Gamma_+ = 0.01$, $\Delta\omega_- = -0.05$, $\Delta\omega_+ = 0.05$, $q_- = -5.1 - 1.0i$, $q_+ = 4.9 - 1.0i$. The approximated result for $|c_1|$ is compared with the exact solution in Fig. 1(a) (dotted lines). It can be seen that the two split peaks are well reproduced by the Fano formula with a complex Fano parameter. Similarly, the approximated result for $|c_2|$ is shown in Fig. 1(b). The widths of the resonance peaks and the resonance frequencies have the same value in $|c_1|$ and $|c_2|$. However, the values in the frequency region between the peaks is smaller in $|c_1|$ because the peaks in $|c_1|$ are due to the Fano resonance, whereas the peaks in $|c_2|$ are due to the Lorentz resonance.

The system examined in the present paper is very simple but rich in the physical content of resonance. Our formulas are not only helpful for understanding Fano resonance in this system but is also expected to be useful in considering other physical systems, because the Fano resonances are one of the general wave phenomena. In particular, these are expected to be useful for controlling the line shape of Fano resonance.

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