

# Topology optimization of large ultrasonic tools for uniform vibration

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## 1. Introduction

To make uniform vibration distribution of ultrasonic bonding tools is vital for industrial applications. The shape design is complicated and requires a lot of know-how due to its wave nature [1]. Topology optimization is a structural optimization method that determines the arrangement of components and materials, including voids, to maximize or minimize the objective function within a design space [2,3]. In this study, we aim to establish a methodology for designing vibratory tools through topology optimization.

## 2. Objective function of topology optimization

**Fig. 1** shows the flowchart of topology optimization in this paper. Each element has a design variable of density  $\rho$  ( $0 < \rho \leq 1$ ), the intermediate state between void and base material. The finite element analysis is repeated to find the optimal shape by changing the density distribution.

The stiffness of each element depends on the interpolation technique of different density methods. The rational approximation of material properties (RAMP) method has the advantage of stable acquisition of vibration mode. Young's modulus  $E$  and mass density  $\rho_m$  are expressed as

$$E = \frac{\rho E_0}{1 + q(1 - \rho)}, \quad \rho_m = \rho \rho_0, \quad (1)$$

where  $E_0$  and  $\rho_0$  are Young's modulus and mass density of base the material, respectively.  $q=3$  is used in this study.

Objective function proposed in this paper is

$$\Theta = \sum_i \text{Re} [P_i^* Q_i], \quad (2)$$

$$P_i = \mathbf{f}^T \mathbf{u}_i, \quad Q_i = \mathbf{g}^T \mathbf{u}_i, \quad (3)$$

where  $\mathbf{f}$  and  $\mathbf{g}$  are the input force from the ultrasonic transducer and virtual force to detect the target point vibration, respectively. The vibration distribution  $\mathbf{u}_i$  from the mode  $i$  is expressed using modal superposition method using eigenvector  $\boldsymbol{\phi}_i$  as

$$\mathbf{u}_i = \frac{(\mathbf{f}^T \boldsymbol{\phi}_i) \boldsymbol{\phi}_i}{\boldsymbol{\phi}_i^T [-\omega^2 \mathbf{M} + j\omega \mathbf{C} + \mathbf{K}] \boldsymbol{\phi}_i}, \quad (4)$$

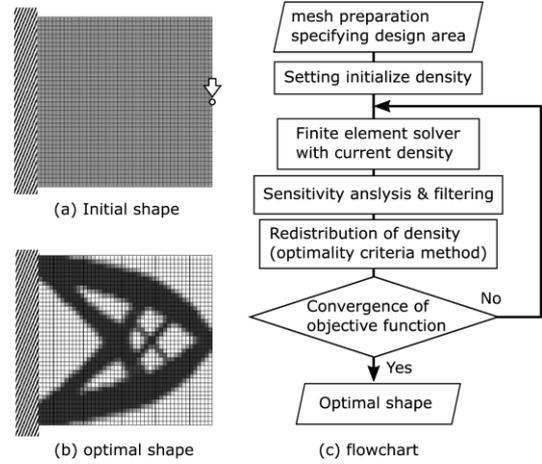


Fig. 1 Concept of topology optimization and its flowchart.

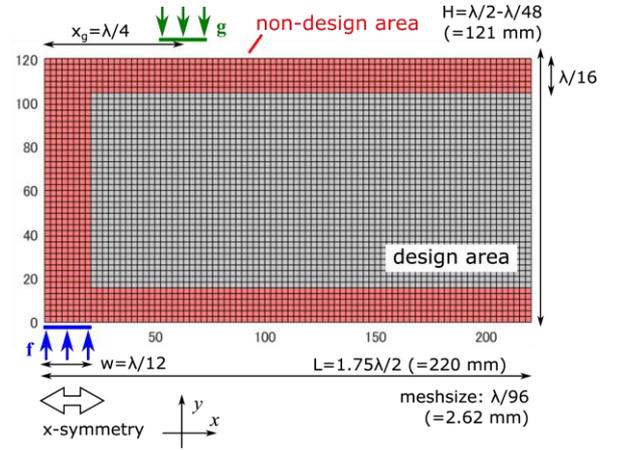


Fig. 2 Calculation model and design area for topology optimization.

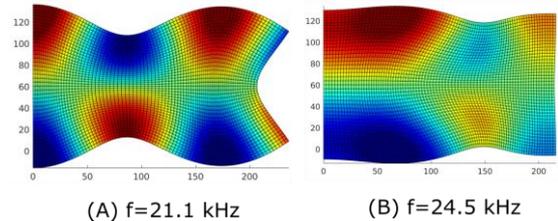


Fig. 3 Vertical displacement distribution of typical modes in initial density distribution.

where  $M$ ,  $C$ , and  $K$  are mass, damping, and stiffness matrix.  $\omega$  is the angular frequency of the transducer. The density distribution is optimized using optimality criteria method using sensitivity  $\partial\Theta/\partial\rho$ .

### 3. Calculation model and design area

**Fig. 2** shows the design area of the ultrasonic tool, which is a half model of the tool by Adachi *et al.* [1] excluding the slit and trap horn. The vibration input  $f$  at 20 kHz is supplied at the center of the lower surface of the tool. The detecting force  $g$  for uniform vibration on the top surface is located at position 1/4 from the center, which tends to be in the opposite phase of the vibration input.  $\lambda = f^{-1} \sqrt{E_0 / \rho_0}$  is the wavelength of the longitudinal wave. The base material is aluminum alloy with Young's modulus  $E_0=68.5$  GPa, Poisson's ratio is 0.34, and material density  $\rho_0=2700$  kg/m<sup>3</sup>. The region is divided by 1/96, and ten elements from the center and six elements on the top and bottom surfaces are assumed to be non-design elements. The mode superposition is assumed to be a mode that exists between 15 and 25 kHz.

**Fig. 3** shows the typical vibration mode in initial density distribution. Mode (A) is the nearest mode of vibration frequency of 20 kHz; therefore, Mode (A) is dominant in the initial vibration distribution  $\mathbf{u} = \sum_i \mathbf{u}_i$ . (B) is promising for maximizing the objective function because  $P_i$  and  $Q_i$  are in phase.

### 4. Optimal shape and objective function history

**Fig. 4(a)** shows optimal shape, density distribution, after optimization. The black and white area indicate the base material and void. The final volume fraction is 0.95. **Fig. 4(b)** shows the vibration distribution with final density distribution. Although the vibration on the output surface are not completely uniform, but the objective of the optimization is fulfilled because the  $P_i$  and  $Q_i$  are in phase.

**Fig. 5(a)** shows objective function history.  $\Theta$ , negative in the early optimization stage, has become sufficiently larger than the initial value, although oscillations originate from resonance make the objective function value unstable. **Fig. 5(b)** shows eigenfrequency history. Mode (A), which shows the opposite phase in  $P_i$  and  $Q_i$ , moves away from the drive frequency, and the useful mode (B) approaches the drive frequency.

### 5. Conclusion

Topology optimization was formulated to obtain the uniform vibration distribution over the ultrasonic vibration tool's output surface. The optimal shape with a hole near the center of the tool is suggested, resulting in a vibration mode in which the output surfaces vibrate in the same phase.

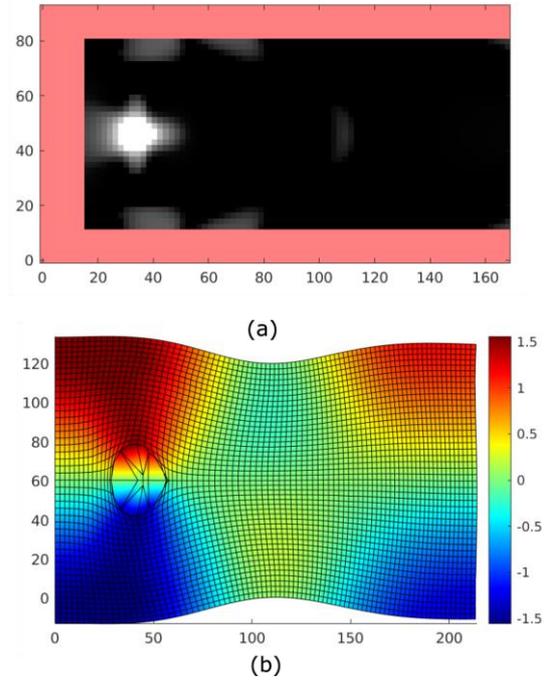


Fig. 4 (a) final density distribution and (b) vibration distribution with final density

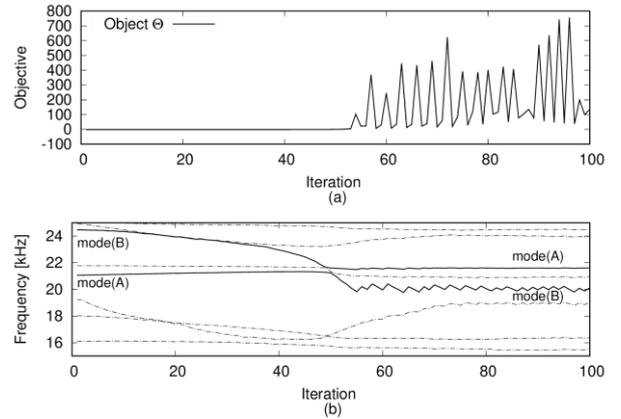


Fig. 5 (a) Objective function history and (b) eigen frequency history.

### Acknowledgment

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### References

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