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Temporal wave control using a Shive wave machine

Motonobu Tomoda[1†], Tetsu Omiya[1], Hayato Takeda[1], Osamu Matsuda[1], and Oliver B. Wright[1] ([1]Grad. School Eng., Hokkaido Univ.)

1. Introduction

Waves can reflect and transmit at the boundaries of two homogenous media. The laws of reflection and transmission are calculated from wave equations and boundary conditions based on continuous physical parameters. For example, the Fresnel equation in optics can be derived from the continuity of the tangential components of the electric and magnetic fields at boundaries. The above discussion applies to the case of spatial boundaries. What happens to waves at temporal boundaries? In fact, waves also reflect and transmit at temporal boundaries. To make temporal boundaries, however, the physical parameters of a media should be changed suddenly (on a scale much faster than the time for propagation through the region of study). Because of the difficulty of making such clear temporal boundaries, there are few experiments to directly investigate the reflected waves at temporal boundaries. An experiment with temporal boundaries on a water surface using a change in the effective gravitational constant has been reported [1].

Here we realize imaging of reflection and transmission at temporal boundaries for the case of a simple and readily visible one-dimensional system. The key idea is to make the wave speed change suddenly by use of a Shive wave machine [2]. This allows the investigation of the wave propagation with the naked eye or by use of a camera, using a system constructed with inexpensive mechanical parts.

2. Theories

2.1 Wave machine theory

Shive wave machines are characterized by the moment of inertia of each rod I and the torsional spring constant κ [3]. It is useful to consider a simple analogy, i.e., the equation of motion of the s-th unit in a conventional one-dimensional mass-spring model (see **Fig. 1**):

$$m\frac{d^2x_s}{dt^2} = k(x_{s+1} + x_{s-1} - 2x_s), \qquad (1)$$

where m is the mass, k is the spring constant, and x_s is the displacement of the s-th mass. On the other hand, the equation of motion of the s-th unit

e-mail address: mtomoda@eng.hokudai.ac.jp

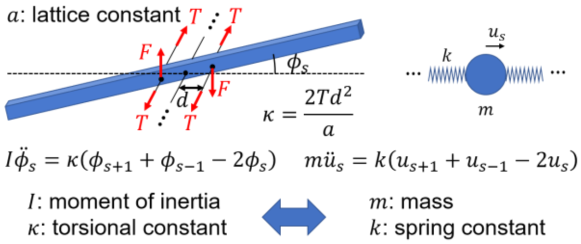

Fig. 1 The correspondence between a wave machine and a mass-spring model.

in a three-wire type Shive wave machine is governed by the equation

$$I\frac{d^2\phi_s}{dt^2} = \kappa(\phi_{s+1} + \phi_{s-1} - 2\phi_s), \qquad (2)$$

where $\kappa = 2Td^2/a$ is the torsional spring constant, T is a tension of the wires, d is the distance from the center of the rods to the wires, a is the lattice constant, and ϕ_s is the angle of the s-th rod from the equilibrium position. One can appreciate the exact correspondence with the one-dimensional mass-spring model. These models are both described by a discretized one-dimensional wave equation.

2.2 Generalized one-dimensional wave equation

The one-dimensional wave equation for a continuous string includes only two physical parameters, i.e., density ρ and tension T of the string. Let us consider the case when both parameters ρ and T depend not only on the position but also on the time. The generalized one-dimensional wave equation becomes

$$\frac{\partial}{\partial t}\left(\rho\frac{\partial u}{\partial t}\right) = \frac{\partial}{\partial x}\left(T\frac{\partial u}{\partial x}\right), \qquad (3)$$

where $u(x,t)$ is displacement. The left-hand side contains a time-derivative of the momentum density, and the right-hand side contains a spatial derivative of the stress.

2.3 Reflection and transmission coefficients at a spatial boundary

Before considering a temporal boundary, let us consider the familiar spatial boundary. Since displacement and stress must be continuous at the boundary between the two regions, we can obtain the reflection and transmission coefficients from the following equation (3):

$$r_{\text{space}} = \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}, \tag{4}$$

$$t_{\text{space}} = \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2}, \tag{5}$$

where $c_i = \sqrt{T_i/\rho_i}$ is the wave velocity and subscript i refers to region 1 or 2. On reflection the frequency is unchanged, and the total wave energy is conserved.

2.4 Reflection and transmission coefficients at a temporal boundary

Next, consider a temporal boundary. In this case, displacement and momentum must be continuous. we can get the reflection and transmission coefficients from equation (3) in the following form:

$$r_{\text{time}} = \frac{\rho_2 c_2 - \rho_1 c_1}{2\rho_2 c_2}, \tag{6}$$

$$t_{\text{time}} = \frac{\rho_1 c_1 + \rho_2 c_2}{2\rho_2 c_2}. \tag{7}$$

On reflection the wavelength and wavenumber are unchanged, but the total wave energy is not conserved because of the changing medium.

3. Experiment

To realize wave reflection phenomena at a temporal boundary, we construct a Shive wave machine that uses 5 wires and 95 rods (**Fig. 2**). The central wire goes through the center of gravity of each rod and supports the rods. Two inside wires are arranged symmetrically about the center, which allows torsional motion to be transferred from rod to rod as the wave propagates. Two outside wires, with a similar function, are also arranged symmetrically, and can be pulled by a stepping motor controlled by a computer. The rods are square cross-section brass bars (320×6×6 mm) arranged with a 20-mm lattice constant. The motion of the wave machine is recorded by four cameras by means of a commercial motion-capture system (OptiTrack).

We have captured reflected waves at a temporal boundary. We have also realized a temporal antireflection layer [4].

4. Conclusion

In conclusion, we demonstrate that it is possible to make a movie of a temporal boundary with a Shive wave machine and to image the reflection of the waves at such a boundary. In future, it should be possible to demonstrate the wave physics of time-dependent media and space-time metamaterials, for example by involving phenomena such as amplification, wave shaping or frequency transformation [5].

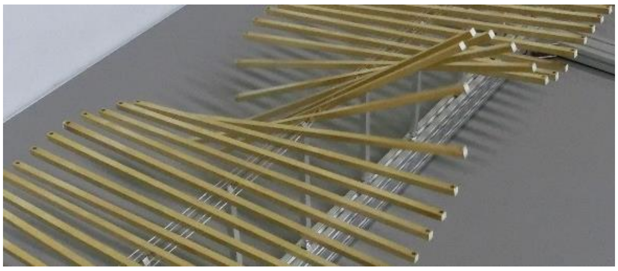

Fig. 2 Photograph of the wave machine.

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