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Three-dimensional analysis of reflection characteristics of Lamb waves at an adhesively bonded stiffener in a plate

Junya Toyota[‡], Naoki Mori, and Takahiro Hayashi (Grad. School Eng., Osaka Univ.)

1. Introduction

Lamb waves can be used for the evaluation of adhesive joints in plate structures. In the previous study[1], transverse stiffnesses of single lap adhesive joints were estimated by measuring the transmission spectra for the normal incidence of the lowest-order antisymmetric (A0) Lamb wave. However, the proposed method is applicable only to the estimation of the stiffness along the wave propagation direction. Oblique incidence of the A0 mode on the joint is expected to be effective to extend the stiffness evaluation method, which requires three-dimensional analysis. To this purpose, the present study aims to clarify reflection characteristics of Lamb waves at an adhesively bonded stiffener in a plate by three-dimensional numerical analysis.

2. Numerical Model and Method

In this study, we analyzed the guided wave propagation in a plate by three-dimensional finite element (FE) analysis in the frequency domain. As shown in **Fig. 1**, a 2 mm thick aluminum plate (plate 1) and a 2 mm thick aluminum stiffener (plate 2) are joined with an adhesive. Absorbing regions surround the plates to suppress the reflections from the edges. The *x, y,* and *z*-axis are taken in the width, length, and thickness directions, respectively. The length of the stiffener is denoted as L.

A single frequency A0 Lamb wave is selectively excited by applying the surface traction

$$\sigma_{zz} = D(x,y)\exp\left[-\mathrm{i}k_{\mathrm{A0}}\{(x-x_{\mathrm{c}})\sin\theta + (y-y_{\mathrm{c}})\cos\theta\}\right] \quad (1)$$

centered at $(x_{\mathrm{c}}, y_{\mathrm{c}})$, where i is an imaginary unit, k_{A0} is the wavenumber of the A0 mode, θ is the angle between the direction of the incident wave and the *y*-axis. The function $D(x,y)$ is associated with the size of the wave source, as shown in Fig. 1.

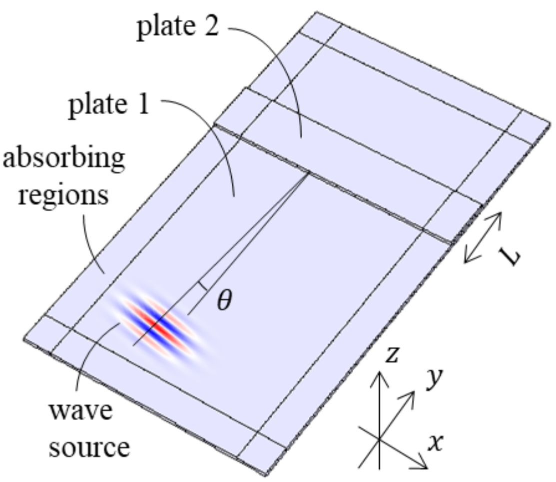

Fig. 1 Numerical model of a plate with an adhesively bonded stiffener.

Based on the previous study[2], the adhesive layer was modeled as a linear spring-type interface. When the displacements and tractions on the bonding surfaces are expressed as $\boldsymbol{u_1} = (u_{1x}, u_{1y}, u_{1z})^T, \boldsymbol{T_1} = (T_{1x}, T_{1y}, T_{1z})^T$ for plate 1 and $\boldsymbol{u_2} = (u_{2x}, u_{2y}, u_{2z})^T, \boldsymbol{T_2} = (T_{2x}, T_{2y}, T_{2z})^T$ for plate 2, respectively, these quantites are related by

$$\begin{pmatrix} T_{1x} \\ T_{1y} \\ T_{1z} \end{pmatrix} = -\begin{pmatrix} T_{2x} \\ T_{2y} \\ T_{2z} \end{pmatrix} = \begin{pmatrix} K_{Tx}(u_{2x}-u_{1x}) \\ K_{Ty}(u_{2y}-u_{1y}) \\ K_L(u_{2z}-u_{1z}) \end{pmatrix} \quad (2)$$

at the joint, where $K_{\mathrm{T}x}$ and $K_{\mathrm{T}y}$ are the transverse stiffnesses of the adhesive layer in the *x* and *y*-directions, respectively, and K_{L} is the longitudinal stiffness in the *z*-direction.

The stiffnesses are determined based on Ref. 2. Let the thickness of the adhesive layer, mass density, longitudinal wave velocity, and transverse wave velocity be h, ρ, c_{L}, and c_{T}. If h is sufficiently small and the effect of the adhesive interfaces is neglected, the joint stiffnesses are given by

$$K_{\mathrm{L}} = \frac{\rho c_{\mathrm{L}}^2}{h}, \qquad K_{\mathrm{T}x} = K_{\mathrm{T}y} = \frac{\rho c_{\mathrm{T}}^2}{h} \quad (3)$$

Assuming a 10 μm thick epoxy adhesive layer, the above stiffnesses are calculated as $K_L = 0.98$ [GPa/μm], $K_{\mathrm{T}} = 0.19$ [GPa/μm]. Based on this estimate, the analysis was performed for the following four cases listed in **Table 1.**

By three-dimensional FE analysis, the out-of-plane displacement on the surface of plate 1 $u_z(x,y)$ is obtained as shown in **Fig. 2**. A two-

Table 1 Model stiffnesses used in calculation

	K_{L} [GPa/μm]	$K_{\mathrm{T}x}$ [GPa/μm]	$K_{\mathrm{T}y}$ [GPa/μm]
(A)	1.00	0.20	0.20
(B)	1.00	0.02	0.02
(C)	1.00	0.20	0.02
(D)	1.00	0.02	0.20

dimensional spatial Fourier transform

$$A(k_x, k_y) = \iint_{-\infty}^{\infty} u_z(x, y) \exp\{\mathrm{i}(k_x x + k_y y)\} dxdy \quad (4)$$

is performed to extract the spectra of the incident and reflected A0 mode waves. The magnitude of the wavenumbers of both waves is the same, but the signs of the wavenumber components in the *y*-direction are opposite. When the spectra of the incident and reflected A0 modes are expressed as I_{A0} and B_{A0}, respectively, the reflection coefficient of the A0 mode is calculated as

$$R_{\mathrm{A0}} = \frac{B_{\mathrm{A0}}}{I_{\mathrm{A0}}} \quad (5)$$

The above calculations are performed for different frequencies to obtain the relationship between the frequency and the reflection coefficient.

3. Results and Discussions

In the case of $L = 30\,[\mathrm{mm}]$ and $\theta = 10\,[\mathrm{deg.}]$, the numerical results of the reflection coefficient for a frequency range 200 – 300 kHz are shown in **Fig. 3**. In all cases (A) - (D), the amplitude reflection coefficient $|R_{\mathrm{A0}}|$ takes multiple extreme values. These frequencies are called extreme frequencies in this study. Multiple reflected waves exist in the adhesively bonded stiffener, and the interference of these waves can weaken or strengthen $|R_{\mathrm{A0}}|$ at different frequencies. This leads to multiple extreme values of $|R_{\mathrm{A0}}|$.

The comparison of (A) and (B) shows that as the transverse stiffnesses $K_{\mathrm{T}x}$ and $K_{\mathrm{T}y}$ decreases, the extreme frequencies decrease. This tendency can be seen in the results for (A) and (C), which show that when only the transverse stiffness in the *y*-direction $K_{\mathrm{T}y}$ is decreased, the extreme frequencies of (C) become close to those of (B). The comparison between (A) and (D) shows that the extreme frequencies become low when the transverse stiffness in the *x*-axis direction becomes smaller, but the difference from (A) is not significant compared to the other two cases.

The above results show that the extreme frequencies are more sensitive to changes in $K_{\mathrm{T}y}$ than in $K_{\mathrm{T}x}$. This would be attributed to the propagation direction of the Lamb wave in the joint. The A0-type mode propagating in the overlap region is affected by the transverse stiffness along the propagation direction.[2] In the present analysis, since the incident angle is $\theta = 10\,[\mathrm{deg.}]$, the propagation direction in the bonded region is approximately 12 – 14 deg. Therefore, the contribution of $K_{\mathrm{T}y}$ to the changes in the extreme frequencies would be more significant than that of $K_{\mathrm{T}x}$.

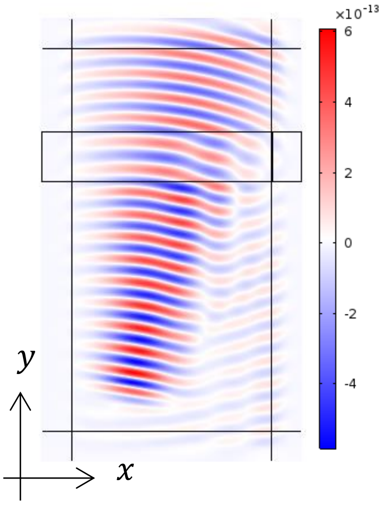

Fig. 2 Out-of-plane displacement calculated on the surface of plate 1.

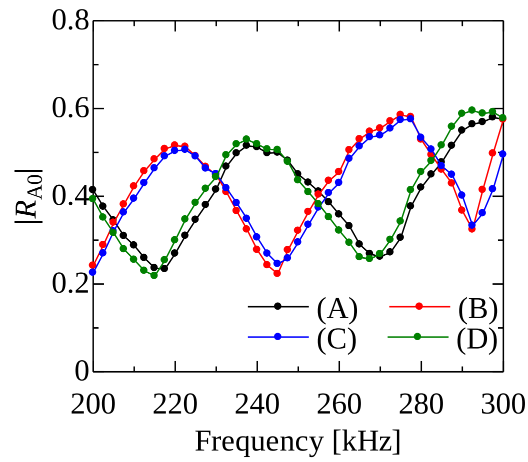

Fig.3 Relationship between $|R_{\mathrm{A0}}|$ and frequency for different stiffnesses.

Acknowledgments

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References

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