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Energy trapping of in-plane vibration in a hollow cylinder with a circumferential groove on the inner surface

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1. Introduction

QCM (Quartz Crystal Microbalance) is used to detect minute substances such as proteins. In this device, the energy trapping phenomenon of elastic waves in a crystal plate is utilized[1,2]. The plate thickness increases at the electrode part on the piezoelectric plate, and the energy trapping phenomenon occurs due to the shape change. There is a need to improve the sensitivity of QCM for the detection of very small amounts of substances.

Similar trapping phenomena have been confirmed not only in flat plates but also in round bars and hollow cylinders. Johnson *et al.*[3] showed that surface waves propagating in the circumferential direction of a round bar are trapped at the stepped part. Hayashi[4] has formed a thin groove in a cylindrical structure and confirmed numerically and experimentally the energy trapping of out-of-plane vibration in the groove of the cylinder.

Here, we assume application as a sensor by filling the inside of the cylinder with a solution of minute substances, adsorbing the substance on the inner surface of the groove, and measuring the frequency change of the vibration in the groove. Since the vibration energy is confined in the thin groove, it is expected to improve the detection sensitivity of the adsorbed substance.

The purpose of this study is to research the energy trapping phenomenon of in-plane vibration generated in a hollow cylindrical groove and to examine its applicability to sensors.

2. Approximate solution of resonance frequency caused by circumferentially propagating in-plane vibration waves

We assume that a wave vibrating in the longitudinal direction propagates circumferentially in a hollow cylinder of inner diameter $2a$ and outer diameter $2b$. As shown in **Fig. 1**, we assume that this wave propagates along a line with a representative radius r_0 $(a \leq r_0 \leq b)$ at a shear wave velocity of c_T. The phase match condition of the circumferential wave is written as the relationship between wavelength and the circumference of the pipe as,

$$2\pi r_0 = n\lambda, \tag{1}$$

where λ is the wavelength and n is a positive integer. This is the resonance condition for waves propagating in the circumferential direction.

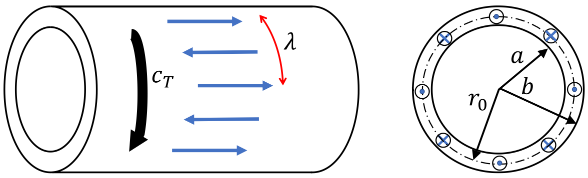

Fig. 1 Hollow cylinder of inner diameter $2a$ and outer diameter $2b$.

When assuming a hollow cylindrical structure with a groove on the inner surface as shown in **Fig. 2**, the representative radius r_0 becomes larger at the groove, and accordingly the resonance wavelength λ becomes larger. On the other hand, resonance does not occur in the non-groove area at the same frequency.

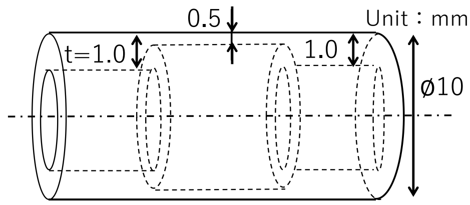

Fig. 2 Hollow cylinder with a groove on the inner surface.

3. Analysis of energy trapping by dispersion curves

To consider the energy trapping phenomenon when in-plane vibration is excited to the groove, we use dispersion curves of the longitudinally propagating guided wave.

Fig. 3 shows the dispersion curves for guided waves propagating in the longitudinal direction of a hollow nickel alloy cylinder (longitudinal wave velocity 5640 m/s, shear wave velocity 2970 m/s), expressing frequency dependence of the longitudinal wavenumber. Only modes with circumferential order $n = 6$ are shown. The vertical axis is the frequency, and the right and left sides of the horizontal axis are the real part $\mathrm{Re}(k)$ and the imaginary part $\mathrm{Im}(k)$ of the wave number k, respectively. The red lines show the dispersion curves for a cylinder with the outer diameter of 10 mm and the thickness of 0.5 mm, such as the groove in Fig. 2. The black lines show dispersion curves of a cylinder with the outer diameter of 10 mm and the thickness of 1 mm, such as the non-groove area in Fig. 2. The solid line indicates the real part of the wavenumber, and the

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dashed line indicates the imaginary part of the wavenumber. The displacement distribution of the guided wave in the longitudinal (z) direction is expressed by $\exp(ikz)$, and if the wavenumber k is a real number, the guided wave continues to propagate without damping. On the other hand, if the wave number k is a purely imaginary number, the displacement damps exponentially and the guided wave does not propagate.

The frequency for $k = 0$ is called the cutoff frequency, which means that the wavelength becomes infinite (uniform displacement distribution) in the longitudinal direction. The cutoff frequency for the groove area in Fig. 2, which is 10 mm in outer diameter and 0.5 mm in thickness, is 595 kHz, and the cutoff frequency for the non-groove area in Fig. 2, which is 10 mm in outer diameter and 1 mm in pipe thickness, is 630 kHz. Considering the displacement distribution $\exp(ikz)$ of the guided wave, when wave is excited at 595-630 kHz to the groove, the waves propagate without damping in the groove area, while they are damped and do not propagate in the non-groove area. In other words, it is predicted from dispersion curves that the energy trapping occurs in the groove.

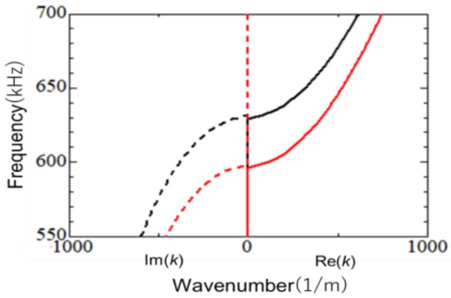

Fig. 3 Dispersion curves of longitudinally propagating guided waves.

4. Numerical calculation

Since the dispersion curves used in Sect. 3 are obtained by using a hollow cylinder with infinite length, direct analysis is required using finite element method for the energy trapping frequency in the groove in a hollow cylinder with a finite length in the longitudinal direction. In this section, the calculations were performed using COMSOL Multiphysics®.

The groove area was 10 mm in outer diameter, 0.5 mm in thickness, and 30 mm longitudinally, while the non-groove area was 10 mm in outer diameter, 1 mm in thickness, 70 mm longitudinally, and total longitudinal length is 170 mm. **Fig. 4** shows the result of the longitudinal displacement distribution at a certain instant in color, with the red and blue areas representing the antinodes and the green areas between them are the nodes. This mode has a circumferential order $n = 6$ and a standing wave of vibration was found to appear in the groove, whose frequency was calculated to be 606 kHz. This result satisfies the frequency range of 595-630 kHz predicted from the dispersion curves in Sect. 3.

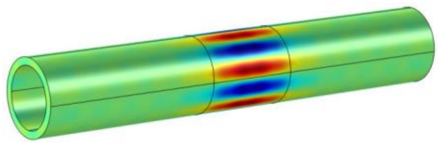

Fig. 4 Longitudinal displacement distribution.

5. Experiment

To experimentally confirm energy trapping in the pipe groove, a hollow cylinder with a circumferential groove on the inner surface was made. The outer diameter, thickness, and length dimensions were the same as in Sect. 4, and a ferromagnetic material, nickel alloy, was used. The groove was excited by electromagnetic acoustic transducer and the vibration at the groove was recorded after the excitation wave. The excitation frequency was set to be 606 kHz based on the results of numerical calculation.

The frequency spectrum of the obtained waveform is shown in **Fig. 5**, indicating the resonance at 626.5 kHz and the energy trapping at the groove, which agree well with the prediction from the dispersion curves in Sect. 3.

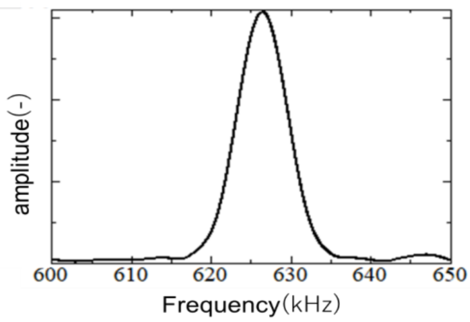

Fig. 5 Frequency spectrum.

6. Conclusion

In this study, we predicted the existence of the energy trapping due to circumferential resonance in the groove using dispersion curves, and calculated its vibration mode and resonance frequency numerically. The resonance frequency was measured and the energy trapping in the groove was confirmed experimentally.

References

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