3Pa2-9

Study of classification of guided wave propagating in cylindrical pipe

Harumichi Sato† (AIST)

1. Introduction

Cylindrical pipes are widely used in applications such as nuclear power plants and micro total analysis systems (μTAS). Nondestructive evaluation (NDE) of such pipes is therefore crucial. NDE as well as ultrasonic flowmeters can be used to characterize pipes filled with fluid. Guided wave propagating in hollow pipes was investigated theoretically by Gazis [1]. Modes of guided waves are classified as longitudinal [L(0,m)], flexural [F(n,m)], and torsional [T(0,m)], where n and m are the circumferential and radial mode parameters, respectively. Nishino *et al.* investigated modal analysis of hollow cylindrical guided waves and proposed that the n-parameter of the T-mode was not limited to zero [e.g., T(1,m) or T(2,m)] [2]. More recently, there have been two ways of classification of modes of the guided waves. In this article, the author attempts to calculate the sound velocities of guided waves carefully and classify them.

2. Theoretical results

Figure 1 shows the theoretical model of a cylindrical pipe and its coordinate system (cylindrical coordinates). Analytical details are available in Ref. 3. The matrix equation is obtained from the boundary conditions of $r = a$ and $r = b$. The size of the matrix is 6×6 for a hollow pipe. The phase velocities of guided waves are calculated by the determinant of matrix, and displacements are calculated by the solution of the matrix equation.

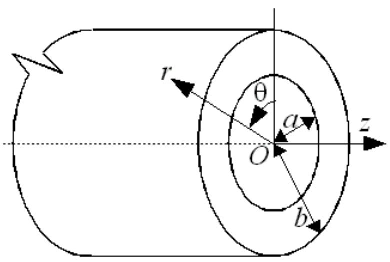

Fig. 1 Theoretical model.

For $n = 0$, the displacements of the pipe are in axial symmetry, and for $n = 1$, the displacements are in plane symmetry. When $n = 0$, the matrix splits into two matrixes: the size of one is 4×4 for L-mode and the other's is 2×2 for T-mode [1]. Therefore, L-mode and T-mode are separated completely. **Figure 2** shows dispersion curves of L(0,m) and T(0,m). The outer and inner diameters of the pipe are $\phi 34$ mm and $\phi 28$ mm, respectively, and its sound velocities of longitudinal and transverse waves are 5790 m/s and 3100 m/s, respectively. **Figure 3** shows the nearby intersection of L(0,4) and T(0,3). Markers indicated with (•) are the calculated results. Because the phase velocities of L-mode and T-mode are calculated by the different matrix equations, the

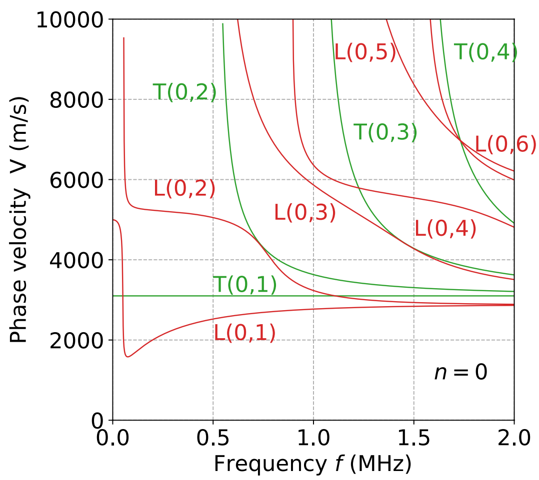

Fig. 2 Phase velocities of L-mode and F-mode.

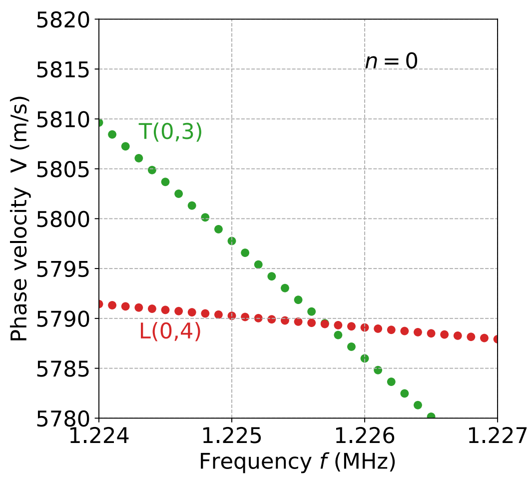

Fig. 3 Phase velocities around the intersection of L(0,4) and T(0,3).

h.sato@aist.go.jp

results are independent and intersect without influencing each other.

Figure 4 shows the dispersion curves for $n = 1$. For example, F(1,2) is named T(1,1) and is close to the sound velocity of transverse waves for $f \to \infty$ according to Ref. 2. However, the author cannot use T(1,m) in this article because F(1,2) and F(1,3) do not intersect and F(1,2) is close to F(1,1) for $f \to 2$ MHz and F(1,3) is close to the sound velocity of transverse waves. F(1,6) and F(1,7), and F(1,8), F(1,9), and F(1,10) also do not intersect. In other words, the author is not able to find an intersect point in **Fig. 4**. **Figure 5** shows the calculated phase velocities (•) of F(1,6) and F(1,7). They are close around 1.226 MHz, but because they do not intersect, the author is not able to index T(1,3) here.

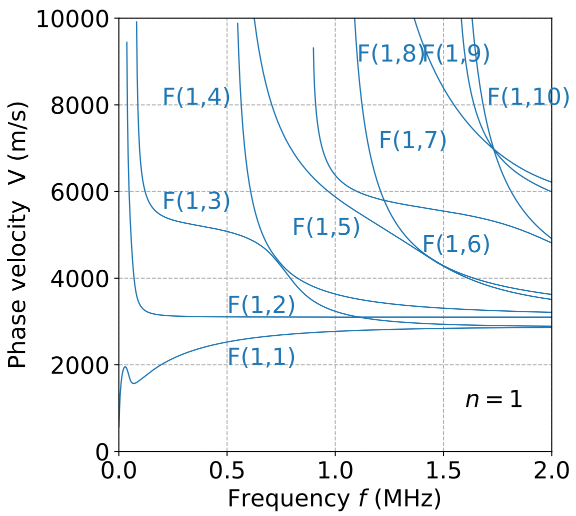

Fig. 4 Phase velocities of F-mode and T-mode.

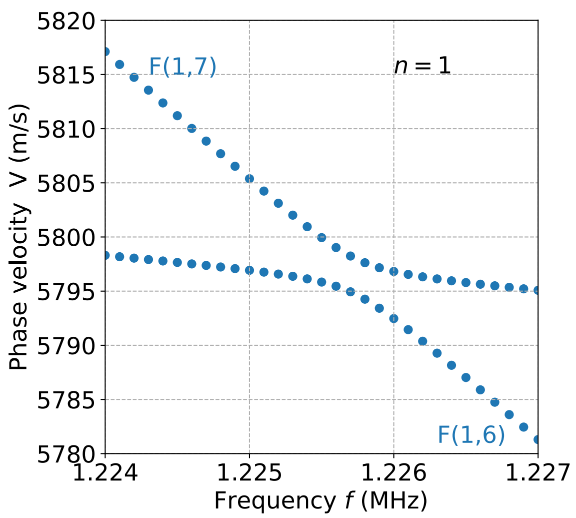

Fig. 5 Phase velocities around the closest point between F(1,6) and F(1,7).

3. Discussions

From Fig. 3, because L-mode and T-mode are separated clearly for $n = 0$, it is easy and reasonable to classify them. However, because F-mode and T-mode are not separated clearly, the author was not able to use the classification method in Ref. 2.

Displacements of a guided wave are expressed as below [3].

$$u_r^{solid} = A(r,n,k,,,)\cos n\theta \cos(\omega t - kz)$$
$$u_\theta^{solid} = B(r,n,k,,,)\sin n\theta \cos(\omega t - kz)$$
$$u_z^{solid} = C(r,n,k,,,)\cos n\theta \sin(\omega t - kz)$$

Here, ω is an angler frequency, k is a wave number, t is time, and A, B, C are represented by using Bessel functions. $u_\theta^{solid} = 0$ and $u_r^{solid} = A\cos(\omega t - kz)$ for $n = 0$, and this result is L-mode. However, $u_\theta^{solid} \propto \sin\theta$ and $u_r^{solid} \propto \cos\theta$ for $n = 1$, and there is no reason u_θ^{solid} or u_r^{solid} must be zero. Therefore, the author considers that the displacements were a mixture of u_θ^{solid} and u_r^{solid} and T(n,m) cannot separate clearly from F(n,m) for $n > 0$.

Group velocity is calculated as

$$v_g = \frac{\partial\omega}{\partial k} \cong \frac{\Delta\omega}{\Delta k} \text{ and } \Delta k = \frac{V_1\omega_2 - V_2\omega_1}{V_1V_2}.$$

Here V_1 and V_2 are phase velocities and those velocities must be selected in the same mode. Therefore, group velocities are affected by the classification of modes. To identify the mode of guided waves in experiments, group velocity is mainly used, so the classification of modes is important, and the author considers that the n-parameter of the T-mode is limited to zero.

4. Conclusions

The author investigated dispersion curves of guided waves carefully and discussed their classification. Because this result of classification and shape of dispersion curves makes the difference of group velocities, the proposed classification method is significant.

Acknowledgment

This work was supported by JSPS KAKENHI Grant Number JP26390109. The author would like to thank Enago (www.enago.jp) for the English language review.

References

1. D. C. Gazis: Jpn. J. Acoust. Soc. Am. **31** (1959) 568.
2. H. Nishino *et al.*: Jpn. J. Appl. Phys. **40** (2001) 364.
3. H. Sato and H. Ogiso: Jpn. J. Appl. Phys. **49** (2010) 07HC08.