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# Two-dimensional displacement estimation using received time distribution of scattered wave on elements in ultrasonic probe

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## 1. Introduction

We have been developing a high-precision measurement method for two-dimensional (2-D) displacement of the myocardium for the evaluation of cardiac function. The received time distribution of scattered waves on the elements in the ultrasonic probe changes by the 2-D displacement of the target [1].

In the present study, we propose the method to estimate the 2-D displacement of the target using this change in the time distribution between frames with a single unfocused wave transmission without scanning the beam.

## 2. Method

2.1 2-D displacement estimation using received time distribution of scattered wave on elements.

In the present method, the change in the received time distribution of scattered waves at the ultrasonic probe elements caused by the 2-D displacement of the target is detected, and the 2-D displacement is estimated from the detected change in the received time distribution. As shown in **Fig. 1**, let the center element position among $K$ elements be origin $(x_0 = 0)$. The time $T(x_k; x_s, z_s)$ when the transmitted plane wave is scattered by the target at position $(x_s, z_s)$ and received by the $k$th element with position $x_k$ is given by

$$T(x_k; x_s, z_s) = \frac{z_s}{c_0} + \frac{\sqrt{z_s^2 + (x_s - x_k)^2}}{c_0}, \quad (1)$$

where $c_0$ is the speed of sound in the medium. When the measurement target displaces from $(x_s, z_s)$ to $(x_s + \Delta x, z_s + \Delta z)$, the time difference distribution $\{\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)\}$ between $\{T(x_k; x_s, z_s)\}$ and $\{T(x_k; x_s + \Delta x, z_s + \Delta z)\}$ is given by

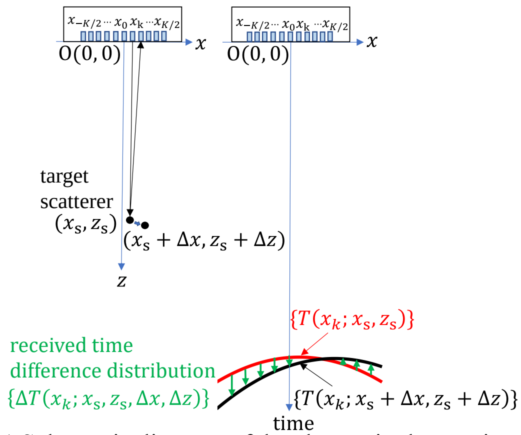


Fig. 1 Schematic diagram of the change in the received time distribution of scattered wave on elements caused by the 2-D displacement of the target.

$$\Delta T(x_k; x_s, z_s, \Delta x, \Delta z) = \frac{\Delta z + \sqrt{(z_s + \Delta z)^2 + (x_s + \Delta x - x_k)^2} - \sqrt{z_s^2 + (x_s - x_k)^2}}{c_0}. \quad (2)$$

To detect the received time difference $\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)$ from the element radiofrequency (RF) signal, the complex cross-correlation $C_n(x_k, \tau; x_s, z_s)$ between frames is calculated for each element using Eq. (3), where $y_{n,k}(t)$ is the RF signal received by the $k$th element in the $n$th frame, $\tau$ denotes the shift of the correlation window, and $w$ denotes the correlation window length. The shift $\hat{\tau}$ of the correlation window which maximizes the cross-correlation $C_n(x_k, \tau; x_s, z_s)$ is determined as the received time difference $\widehat{\Delta T}(x_k)$, as shown in Eq. (4).

The 2-D displacement is determined by searching $(\widehat{\Delta x}, \widehat{\Delta z})$ that minimizes the root mean squared error $RMSE(\Delta x, \Delta z)$ between $\widehat{\Delta T}(x_k)$ and $\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)$ as shown by Eqs. (5) and (6).

$$C_n(x_k, \tau; x_s, z_s) = \frac{\sum_{t=0}^{w} \left[ y_{n,k}^*(T(x_k; x_s, z_s) + t) \cdot y_{n+1,k}(T(x_k; x_s, z_s) + t + \tau) \right]}{\sqrt{\sum_{t=0}^{w} \left| y_{n,k}(T(x_k; x_s, z_s) + t) \right|} \sqrt{\sum_{t=0}^{w} \left| y_{n+1,k}(T(x_k; x_s, z_s) + t + \tau) \right|}}. \quad (3)$$

$$\widehat{\Delta T}(x_k) = \arg\max_{\tau} C_n(x_k, \tau; x_s, z_s). \quad (4)$$

$$(\widehat{\Delta x}, \widehat{\Delta z}) = \arg \min_{(\Delta x, \Delta z)} RMSE(\Delta x, \Delta z). \quad (5)$$

$$RMSE(\Delta x, \Delta z) = \sqrt{\frac{1}{K} \sum_{k=-K/2}^{K/2} \left| \Delta T(x_k; x_s, z_s, \Delta x, \Delta z) - \widehat{\Delta T}(x_k) \right|^2}. \quad (6)$$

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## 3. Experiment

### 3.1 Experimental conditions

A wire in the water tank was moved by $0.2\ \mathrm{mm}$ in the axial direction and $1\ \mathrm{mm}$ in the lateral direction. The displacement of the wire was set considering the maximum displacement of the myocardium between the consecutive frames during the rapid filling phase when the frame rate is $200\ \mathrm{Hz}$. The element RF signals were acquired in the short-axis view of the wire using plane wave transmission by the ultrasound diagnosis apparatus (RSYS-0018, Front-end Technology). The elements with $K = 95$ out of 96 with the element spacing of $0.2\ \mathrm{mm}$ in the sector probe (UST-52101N, ALOKA) were used. The plane wave was transmitted with a center frequency of 3.75 MHz, and a sampling frequency was 62.5 MHz. The shallowest edge of the correlation window was set at the first positive peak of the RF signal amplitude at each element. The length of the window, $w$, was set to $1.2\ \mathrm{mm}$. The shift of the window, $\tau$, was set to $\pm 0.6\ \mathrm{mm}$. For calculating $RMSE(\Delta x, \Delta z)$, a range of $\pm 2.4\ \mathrm{mm}$ was searched at $0.01\ \mathrm{mm}$ intervals in both axial and lateral directions.

### 3.2 Result and discussion

**Figure 2(a)** shows the element signals acquired before displacing the wire and Fig. 2(b) shows that acquired after displacing the position of the wire by $(\Delta x, \Delta z) = (1\ \mathrm{mm}, 0.2\ \mathrm{mm})$. The edge of the correlation window was shown by the red line. **Figure 3(a)** shows the element signals received from the wire at $(x_s, z_s)$, which were delayed by the ideal delay times $\{T(x_k; x_s, z_s)\}$ calculated by Eq. (1). Figure 3(b) shows the element signals after displacing the wire which were delayed by the same ideal delay times $\{T(x_k; x_s, z_s)\}$ as Fig. 3(a). The detected time difference of the element signal between before and after displacing the wire, $\widehat{\Delta T}(x_k)$, was shown by the red line, in Fig. 3(b). As shown in Fig. 3(b), the wavefront of the element signals has a slope that was caused by the 2-D displacement of the wire. **Figure 4** shows the distribution of $RMSE(\Delta x, \Delta z)$ between $\widehat{\Delta T}(x_k)$ and $\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)$. The position where the $RMSE(\Delta x, \Delta z)$ became minimum was shown by the endpoint of the arrow. As shown in Fig. 4, the 2-D displacement was estimated as $(\widehat{\Delta x}, \widehat{\Delta z}) = (0.97\ \mathrm{mm}, 0.21\ \mathrm{mm})$, which well corresponded to the set value of the displacement $(1\ \mathrm{mm}, 0.2\ \mathrm{mm})$.

**Figures 5(a)** and 5(b) show the distribution of $RMSE(\widehat{\Delta x}, \Delta z)$ when Fig. 4 was cut out in the axial direction at the lateral position $\widehat{\Delta x}$, and the distribution of $RMSE(\Delta x, \widehat{\Delta z})$ when Fig. 4 was cut out in the lateral direction at the axial position $\widehat{\Delta z}$, respectively. As shown in Fig. 5, the slope of the distribution of $RMSE(\widehat{\Delta x}, \Delta z)$ (Fig. 5(b)) was

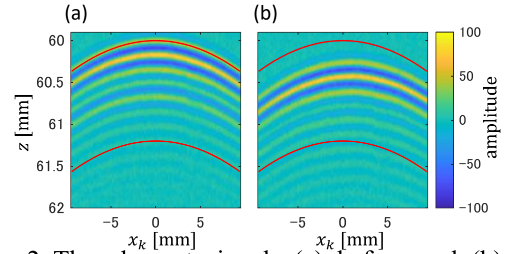


Fig. 2 The element signals (a) before and (b) after displacing the target.

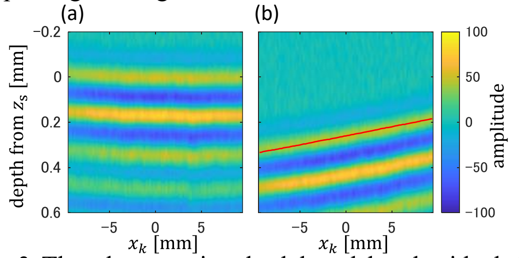


Fig. 3 The element signals delayed by the ideal delay times $T(x_k; x_s, z_s)$ and the received time difference $\widehat{\Delta T}(x_k)$ (red line), (a) before and (b) after displacing the target.

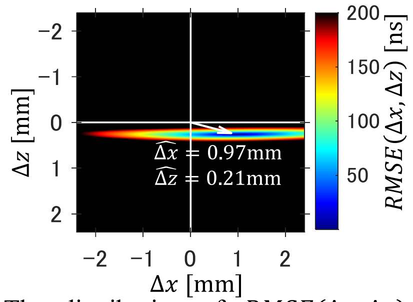


Fig. 4 The distribution of $RMSE(\Delta x, \Delta z)$ between $\widehat{\Delta T}(x_k)$ and $\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)$.

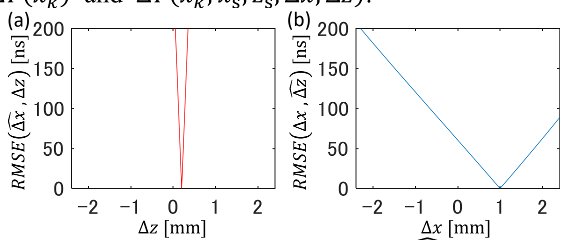


Fig. 5 (a) The distribution of $RMSE(\widehat{\Delta x}, \Delta z)$. (b) The distribution of $RMSE(\Delta x, \widehat{\Delta z})$.

smaller than that of $RMSE(\Delta x, \widehat{\Delta z})$ (Fig. 5(a)), which showed that the changes in the received time difference $\Delta T(x_k; x_s, z_s, \Delta x, \Delta z)$ caused by the lateral displacement $\Delta x$ is small compared to that caused by the axial displacement $\Delta z$.

## 4. Conclusion

In the present study, we proposed the 2-D displacement estimation method using the change in received time distribution of the scattered wave on the elements. In future work, we will examine the proposed method under more complicated case that multiple scatterers exist.

**References**


1. S.-I. Wang *et al.*: IEEE Trans. Ultrason. Ferroelectr. Freq. Control **54** (2007) 70.