

## Mathematical modeling of ultrasonic refraction generated at partially closed crack face

Kazuyuki Nakahata<sup>1†\*</sup>, Kei Onodera<sup>1</sup>, and Taizo Maruyama<sup>2</sup>  
 (<sup>1</sup>Grad. School Sci.&Eng., Ehime Univ.; <sup>2</sup>Grad.School Env.&Soc., Tokyo Tech.)

### 1. Introduction

Nondestructive ultrasonic testing (UT) is widely used to inspect cracks in a structural component. However, when the crack face is contacted, there is a concern of overlooking or underestimating the crack height because ultrasonic waves pass through the crack face. Nonlinear ultrasonic methods<sup>1,2)</sup> have been vigorously studied as a method for evaluating closed cracks. However, the fatigue crack is not entirely closed from the viewpoint of the microscopic aspect. In our experiment using the photoacoustic wave visualization system<sup>3)</sup>, a refracted shear wave generated from the crack interface was confirmed when the oblique incident ultrasonic waves to the fatigue crack. This indicates that ultrasonic waves leak from the face of the crack due to partially contacted parts of the crack face.

In this research, we model this phenomenon with theoretical and numerical approaches. The theoretical analysis of the interface uses a spring model to express the contact stiffness. On the other hand, numerical simulations using the elastodynamic finite integration technique<sup>4)</sup> (EFIT) are applied to demonstrate wave propagation and scattering by the crack. Here, we model the partial contact of the crack face using a small mesh with a two-phase condition.

### 2. Visualization of ultrasonic wave around fatigue crack

A fatigue crack was made by processing a stainless steel SUS304 ingot with a length of 351 mm, that has a rectangular cross-section height of 38 mm and width of 10 mm, as shown in **Fig.1(a)**. The pressure (P) and shear (S) wave velocities are  $c_P=5760$  m/s and  $c_S=3070$  m/s, respectively.

To initiate the closed crack from the center of the bottom surface, we first prepared a slit with a width of 0.3 mm and a depth of 5.0 mm at the bottom surface by the electrical discharge machining. Then we propagated the crack from there by a three-point bending test. After the fatigue crack growth reached 15mm, we stopped the bending test and measured the crack width with a digital microscope. As shown in **Fig.1(b)**, the width near the slit and the crack tip are 0.009 mm and 0.001 mm or less, respectively.

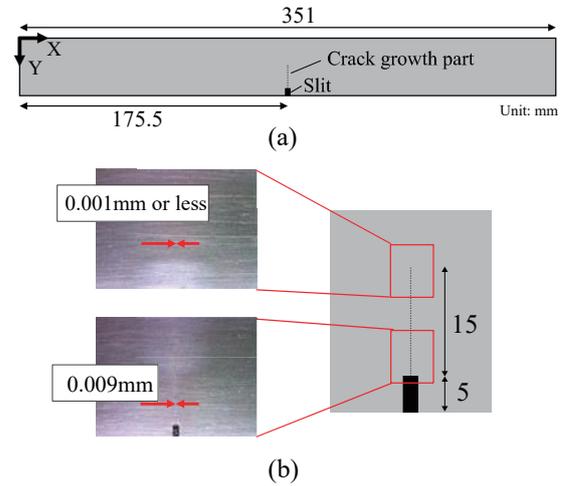


Fig. 1 (a) Stainless steel specimen. (b) Closeup photo of the fatigue crack initiation part.

To investigate the behavior of ultrasonic waves around the fatigue crack, we performed visualization experiments of wave propagation in the stainless steel specimens using the photoacoustic method<sup>3)</sup>. We received ultrasonic wave with a transducer at each point on the surface of the specimen, which were generated by laser irradiation. Here, by using the reciprocity of the elastodynamics problem, the wavefield can be visualized as if the ultrasonic waves were excited from the transducer. Here, the ultrasonic waves were received using an acrylic wedge. The angle of the wedge was set so that the P wave incident angle becomes 51 degrees to the crack face. The laser was scanned with an irradiation pitch of 0.1 mm in the red frame shown in the upper part of **Fig. 2**. The sampling rate was 100MS/s.

The visualization results are shown in **Fig.2(a)-(d)**. We plot the normalized amplitudes by dividing them by the maximum value within whole propagation time. It can be confirmed that the refracted S wave is generated when the incident P wave passes through the crack face. This indicates that ultrasonic waves leak from the crack due to partially contacted parts of the crack face.

Here we check the refracted direction of the S wave. Let  $\theta_S$  be the refraction angle of the refracted S wave. Since the incident angle  $\theta_{in}$  to the crack face is  $51^\circ$ , the following Snell's law shows that  $\theta_S$  is  $24.5^\circ$

$$\frac{\sin \theta_{in}}{c_P} = \frac{\sin \theta_S}{c_S} \quad (1)$$

E-mail: <sup>†</sup>nakahata@cee.ehime-u.ac.jp

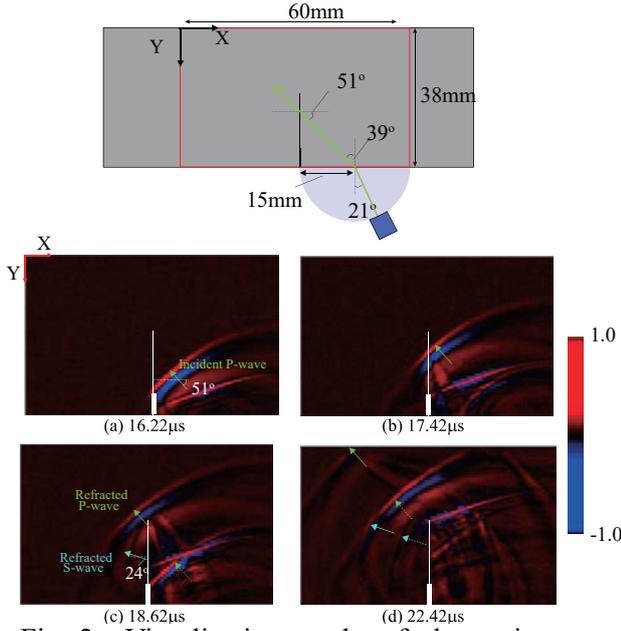


Fig. 2 Visualization results of ultrasonic wave propagation around the fatigue crack.

Since the refraction angle shown in the results in Fig.2 is 24°, it is understood that the experimentally measured refraction angle agrees well with the theoretical calculation by Snell's law.

### 3. Modeling of wavefield by fatigue crack

#### 3.1 Analytical model

The upper figure in Fig. 3 shows that steady-state traveling waves incident two-connected half-spaces. Each space is assumed to be the same isotropic linear elastic body, and the interface is modeled by two linear springs with finite stiffness in the normal and tangential directions. We calculated the reflection ( $R$ ) and refraction ( $T$ ) coefficients in the case of the P-wave incident. The interfacial

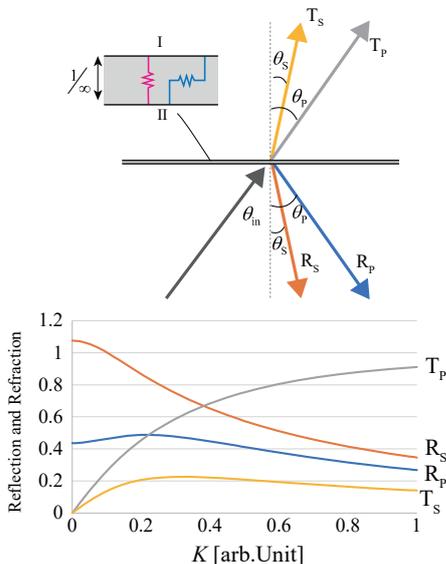


Fig. 3 Reflection and refraction coefficients at plane interface connected with two springs.

connection strengthens when the spring constant  $K$  increases, and cracks become a tightly bonding state. On the other hand, if  $K$  becomes small, the interface stiffness decreases, and the crack goes to the open state. As shown in the bottom figure in Fig. 3, the coefficient  $T_s$  of the refracted S wave becomes maximum at the spring constant  $K$  with about 0.32.

#### 3.2 Numerical model

We used the elastodynamic finite integration technique (EFIT). To express the fatigue crack, we model the partial contact of the crack face using a small mesh with a two-phase condition. For the part of the unbonded crack face, we represent it by a minute cavity cell. On the other hand, we use an elastic body cell for the contact face. Using these cells, we made a fatigue crack whose interface section of 50% was contacted. Figure 4 shows the snapshots of wave propagation with the EFIT calculation. A refracted S wave generated from the crack face by the incident P wave can be confirmed. The direction of the refracted S wave showed good agreement with the calculated result by Snell's law.

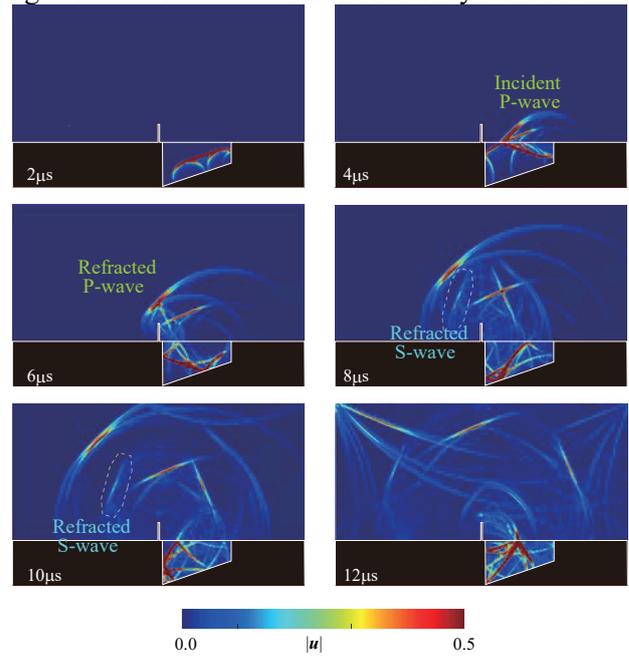


Fig. 4 Visualization results of ultrasonic wave propagation around the fatigue crack.

### 4. Concluding remarks

The comparison of the analytical and numerical models will be done in the next step. The application of the combined model is future work.

#### References

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