

# Consideration of Feedback Mechanism in Depolarizing Field in Dielectric Material and Piezoelectric Transducer

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## 1. Introduction

The depolarizing field strongly affects the characteristics of dielectric and piezoelectric materials and devices.

The permittivity  $\varepsilon$  of a dielectric material should be measured in the environment in which the depolarization coefficient is maximal. In this study, firstly, a feedback system to estimate the permittivity is considered, and by comparing a similar feedback system to estimate the permeability  $\mu$  of a magnetic material, it is clarified that appropriate estimates of  $\varepsilon$  and  $\mu$  can be obtained when no feedback is provided and that what is called  $E$ - $B$  correspondence is more essential than  $E$ - $H$  one.

In a piezoelectric transducer, the depolarizing field causes another effect on the characteristics of mechanical wave, and the behavior of the transducer is classified into two different types: that with longitudinal (L-) effect and that with transverse (T-) effect. In this study, another type of ‘feedback effect’ of depolarizing field is considered, which gives a more reasonable explanation for the characteristics of piezoelectric transducer.

## 2. Feedback system for measurement of permittivity and permeability

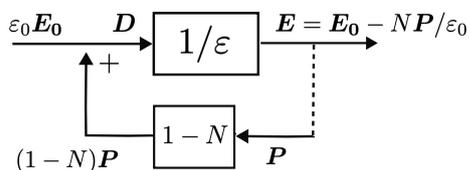


Fig. 1: Feedback diagram for measuring  $1/\varepsilon$ .  $N$  ( $0 < N < 1$ ) is an (averaged) depolarizing coefficient, which affects the measurement via the feedback of depolarizing field from the output (electric field) to the input (dielectric flux density).  $N \rightarrow 0$  for an infinitely long rod, and  $N \rightarrow 1$  for an infinitely thin plate.

Figure 1 shows the effect of feedback of depolarizing field on measuring  $1/\varepsilon$ , where  $\mathbf{E}_0$  is an input electric field,  $N$  ( $0 < N < 1$ ) is an (averaged) depolarizing coefficient,  $\mathbf{P}$  is a generated polarization, and a dielectric flux density  $\mathbf{D}$  is related with a total electric field  $\mathbf{E}$  as  $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$ . A portion of the output is fed back into the input as  $(1 - N)\mathbf{P}$ .

From the closed-loop characteristic, in which the output  $\mathbf{E} \left( = \frac{\mathbf{D} - \mathbf{P}}{\varepsilon_0} \right) = \mathbf{E}_0 - N\frac{\mathbf{P}}{\varepsilon_0}$  and the input  $\mathbf{D} = \varepsilon_0\mathbf{E}_0 + (1 - N)\mathbf{P}$ , an estimated value of  $1/\varepsilon$  is obtained as a function of  $N$ :

$$\frac{1}{\varepsilon}(N) = \frac{E_0 - N\frac{P}{\varepsilon_0}}{\varepsilon_0(E_0 + (1 - N)\frac{P}{\varepsilon_0})}, \quad (1)$$

which provides a correct value of  $1/\varepsilon$  when  $N \rightarrow 1$ ; that is, when no output is fed back into the input.

For comparison, the effect of demagnetizing field on the measurement of a permeability  $\mu$  is schematized in Fig. 2, where  $\mathbf{H}_0$  is an input magnetic field strength,  $N$  ( $0 < N < 1$ ) is an averaged demagnetizing coefficient,  $\mathbf{M}$  is a generated magnetization, and a magnetic field  $\mathbf{B}$  is related with a total magnetic field strength  $\mathbf{H}$  as  $\mathbf{B} = \mu\mathbf{H} = \mu_0(\mathbf{H} + \mathbf{M})$ , where  $\mathbf{H} = \mathbf{H}_0 - N\mathbf{M}$  due to the feedback of  $\mathbf{M}$  by  $-N\mathbf{M}$ . From Fig. 2,  $\mu$  is estimated as

$$\mu(N) = \frac{\mu_0(H_0 + (1 - N)M)}{H_0 - NM}, \quad (2)$$

in which a correct estimate of  $\mu$  can be obtained when  $N \rightarrow 0$  without any feedback effect.

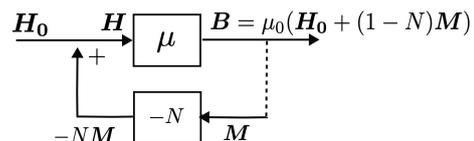


Fig. 2: Feedback diagram to estimate  $\mu$ .

The comparison of the above feedback systems endorses the correspondence of  $\mathbf{E} \leftrightarrow \mathbf{B}$ , rather than that of  $\mathbf{E} \leftrightarrow \mathbf{H}$ , which has often been discussed in the interpretation of electromagnetic theory.

### 3. Influence of depolarization field on piezoelectric transducers

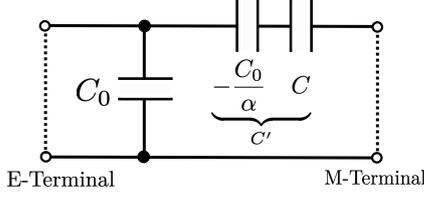


Fig. 3: Equivalent circuit of a piezoelectric transducer at  $\omega \rightarrow 0$ .  $\alpha = 1$  for L-effect and  $\alpha = 0$  for T-effect.

Figure 3 shows the equivalent circuit of a piezoelectric transducer at low frequency limit,  $\omega \rightarrow 0$ , in which  $C_0$  and  $C$  are the dielectric capacitance and the ‘intrinsic’ elastic equivalent capacitance, respectively. The electromechanical coupling coefficient,  $k^2 = C'/(C_0 + C') = (1/C_0)/(1/C_0 + 1/C')$ , is calculated as

$$k^2 = \frac{C}{C_0 + (1 - \alpha)C} \quad (0 \leq k^2 < 1), \quad (3)$$

where  $\alpha = 1$  for L-effect, and  $\alpha = 0$  for T-effect.

The characteristics of piezoelectric transducers can schematically be classified into four patterns by considering whether L- or T-effect works and whether the E-terminal in Fig. 3 is opened or shorted, as shown in Fig. 4.

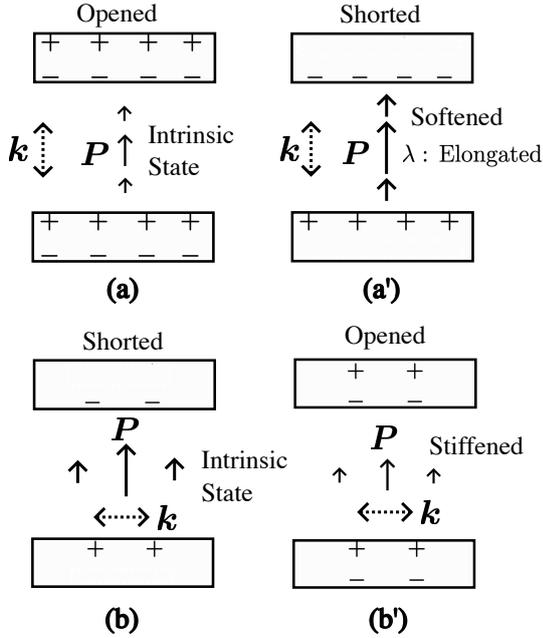


Fig. 4: Schematic diagram of the relationship among the surface charges on electrodes, polarization  $\mathbf{P}$ , and wavenumber vector  $\mathbf{k}$ . (a), (a') L-effect ( $\mathbf{P} \parallel \mathbf{k}$ ) for opened and shorted states, respectively. (b), (b') T-effect ( $\mathbf{P} \perp \mathbf{k}$ ) for shorted and opened states, respectively.

In the L-effect ( $\mathbf{P} \parallel \mathbf{k}$ ), the opened state is regarded as the intrinsic state, as shown in Fig. 4(a), due to the continuity of  $\mathbf{D}$  normal to  $\mathbf{k}$ -planes that are perpendicular to  $\mathbf{k}$ . In the shorted state, Coulomb’s force from the residual charges on the electrodes elongates the wavelength of polarization wave  $\lambda$ , and the elasticity is softened, as shown in Fig. 4(a’), which is represented by  $-C_0$  ( $\alpha = 1$ ) in Fig. 3.

In the T-effect ( $\mathbf{P} \perp \mathbf{k}$ ), the shorted state is regarded as the intrinsic state, as shown in Fig. 4(b), due to the continuity of  $\mathbf{E}$  tangential to  $\mathbf{k}$ -planes. In the opened state, Coulomb’s force from the surplus charges on the electrodes condenses  $\mathbf{P}$  and the elasticity is stiffened, while  $\lambda$  is invariant, as shown in Fig. 4(b’), which is represented by a series connection of  $C_0$  in Fig. 3.

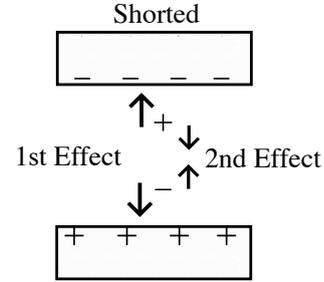


Fig. 5: Primary Coulomb interaction between the surface charges and  $\mathbf{P}$ , and secondary Coulomb interaction inside  $\mathbf{P}$  as a negative feedback process.

As  $\mathbf{P}$  becomes larger, a negative feedback process is expected to occur, as shown in Fig. 5, moderating the decrease of elastic stiffness, especially in the case of a high- $k^2$  material. This process is represented by (I) the depression of the effect of negative capacitance  $-C_0$  in the L-effect and (II) the addition of another positive capacitance in series in the T-effect, represented in common as

$$\alpha \rightarrow \alpha + \Delta\alpha \quad (\Delta\alpha < 0) \quad (4)$$

In the L-effect ( $\alpha = 1$ ), unnaturally, the wavelength  $\lambda \rightarrow \infty$  as  $C \rightarrow C_0$  at the electrical resonance (shorted state), and  $k^2 > 1$  in eq. (3) if  $C > C_0$ . Nature is self-consistent, and the above negative feedback effect keeps the value of  $\lambda$  and  $k^2$  appropriately. (In addition, while the total elastic capacitance is depressed by the effect of  $\Delta\alpha (< 0)$ , the inertial inductance is expected to be enhanced by way of compensation.)