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Anaylsis of characteristic changes depending on the thickness and aspect ratio of the shell in a class-4 flextensional transducer

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1. Introduction

The Class IV Flextensional Transducer is widely used in underwater acoustics, particularly in harbor defense systems, due to its excellent low-frequency characteristics, high electro-acoustic conversion efficiency, and high durability. However, the complex structure that combines longitudinal vibration piezoelectric elements with flexural vibrating metal elements leads to a lack of design flexibility. Therefore, finite element analysis is typically conducted for characteristic analysis, but this approach requires extensive computational resources and time, making it challenging to optimize models or review new designs. G. Brigham proposed a mathematical analysis model, but its complexity reduces practical utility[1)]. Y. Lam suggested a relatively simple analysis model, but it did not sufficiently analyze the characteristics of the Class IV Flextensional Transducer concerning its shape changes[2)].

This study aims to analyze the resonant characteristic changes of a simplified Class IV Flextensional Transducer model based on variations in the thickness and aspect ratio of the elliptical cylindrical shell, thereby identifying design elements and providing foundational data for reviewing new designs.

2 Theoretical model

The computational model of the Class IV Flextensional Transducer used for theoretical analysis is shown in **Fig. 1**. The piezoelectric stack inserted inside the elliptical cylinder is assumed to exert a constant force along the entire length of the cylinder, thus ignoring vibrations in the *z*-direction. Consequently, the vibration displacement of the elliptical cylindrical shell due to the piezoelectric stack's excitation can be considered only in terms of the components normal to the elliptical surface, ξ_N and the tangential components, ξ_T . These components satisfy the following the motion equation:

$$\frac{Y_E h}{R_C}\left(\frac{d\xi_T}{ds}-\frac{\xi_N}{R_C}\right)-\left(\frac{Y_E h^3}{12}\right)\frac{d^3}{ds^3}\left(\frac{d\xi_N}{ds}+\frac{\xi_T}{R_C}\right)=-\omega^2\rho h\xi_N+P_N \quad (1a)$$

$$Y_E h\frac{d}{ds}\left(\frac{d\xi_T}{ds}-\frac{\xi_N}{R_C}\right)+\frac{1}{R_C}\left(\frac{Y_E h^3}{12}\right)\frac{d^2}{ds^2}\left(\frac{d\xi_N}{ds}+\frac{\xi_T}{R_C}\right)=-\omega^2\rho h\xi_T+P_T \quad (1b)$$

Where *ds* is the small length of the arc at the mid-surface of the shell, R_C is the local curvature radius of the mid-surface, *h* is the shell thickness, P_N and P_T are the external normal and tangential pressures on the shell, respectively, ξ_N is the normal displacement function, ξ_T is the tangential displacement function, Y_E is the Young's modulus of the shell, and ρ is the density of the shell.

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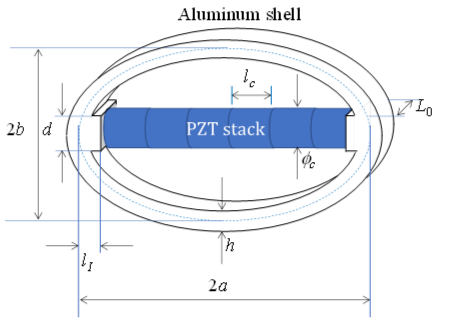


Fig. 1 Calculation model of the class IV Flextensional transducer.

The excitation by the piezoelectric stack is symmetric, so the trial solution for the shell's motion can be expressed as a Fourier series as follows:

$$\xi_N=\sum_{n=0}^{\infty}N_{2n}\cos 2n\theta,\ \ \xi_T=\sum_{n=0}^{\infty}T_{2n}\sin 2n\theta. \quad (2)$$

Where θ is the angle between the shell normal and the *x*-axis. Substituting Eq. (2) into Eq. (1) and organizing them yields:

$$\sum_n N_{2n}\left[\left\{A_{2i,2n}^{N}\right\}^{-1}B_{2i,2n}^{N'}+\left\{A_{2i,2n}^{T}\right\}^{-1}B_{2i,2n}^{T'}-\omega^2\left\{\left\{A_{2i,2n}^{N}\right\}^{-1}C_{2i,2n}^{N}+\left\{A_{2i,2n}^{T}\right\}^{-1}C_{2i,2n}^{T'}\right\}\right]=-\left\{A_{2i,2n}^{N}\right\}^{-1}P_{2i}^{N}-\left\{A_{2i,2n}^{N}\right\}^{-1}P_{2i}^{T}. \quad (3)$$

Here $A_{2i,2n}^{N}=\int_0^{2\pi}\frac{\cos 2n\theta\cos 2i\theta}{R_C}d\theta$,

$$A_{2i,2n}^{T}=\int_0^{2\pi}\frac{2n\sin 2n\theta\sin 2i\theta+\alpha\cos 2n\theta\sin 2i\theta}{R_C}d\theta,$$

$$B_{2i,2n}^{N}=\frac{Y_E h^3}{12}\int_0^{2\pi}\frac{(2n\cos 2n\theta-\alpha\sin 2n\theta)(\alpha\sin 2i\theta-2i\cos 2i\theta)}{R_C^3}\frac{(1-4n^2)i}{n}d\theta,$$

$$B_{2i,2n}^{T}=\frac{Y_E h^3}{12}\int_0^{2\pi}\frac{(2n\cos 2n\theta-\alpha\sin 2n\theta)(\alpha\sin 2i\theta-2i\cos 2i\theta)}{R_C^3}\frac{(4n^2-1)}{2n}d\theta,$$

$$C_{2i,2n}^{N}=\rho h\int_0^{2\pi}R_C\cos 2n\theta\cos 2i\theta\,d\theta,$$

$$C_{2i,2n}^{T}=\rho h\int_0^{2\pi}\frac{R_C\sin 2n\theta\sin 2i\theta}{2n}d\theta,$$

$$P_{2i}^{N}=\int_0^{2\pi}R_C P_N\cos 2i\theta\,d\theta,\qquad P_{2i}^{T}=\int_0^{2\pi}R_C P_T\sin 2i\theta\,d\theta.$$

By using the value of N_{2n} obtained from Eq. (3), the displacement distribution can be determined as follows.

$$\Xi_m=\sum_i N_{2i}S_{2i}, \quad (4)$$

Where $\theta_c=2\tan^{-1}\frac{b-(a-d)}{b+(a-d)}$,

$$S_{2i}=\int_{-\theta_c}^{\theta_c}\frac{\cos\theta\cos 2i\theta+\frac{\sin\theta\sin 2i\theta}{2i}}{\phi_c}R_C d\theta.$$

From the above, the mechanical impedance is as follows:

$$Z_m=\frac{F_c}{u_c}=\frac{F_c}{j\omega\Xi_m}. \quad (5)$$

The equivalent circuit of the Class IV Flextensional Transducer is shown in **Fig. 2**. Here, *p* is the number of piezoelectric elements in the piezoelectric stack, and each component is as follows:

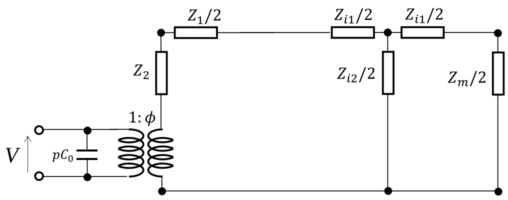


Fig. 2 The equivalent circuit of the Flextensional Transducer.

$Z_1 = jZ_c \tan\frac{pk_c l_c}{2}$, $Z_2 = -j\frac{Z_c}{\sin pk_c l_c}$, $\phi = \frac{-d_{33}(1-k_{33}^2)A}{s_{33}^D\, l_c}$, $Z_c = Z_0\sqrt{1-k_{33}^2}$, $k_c = \frac{\omega}{v\sqrt{1-k_{33}^2}}$, $v = \frac{1}{\sqrt{s_{33}^D \rho_c}}$, $k = \frac{\omega}{c}$, $A = \pi\left(\frac{\phi_c}{2}\right)^2$, $Z_{i1} = jZ\tan\frac{kl_I}{2}$, $Z_{i2} = -j\frac{Z}{\sin kl_I}$, $Z = \rho c A$, $c = \sqrt{\frac{Y_E}{\rho}}$.

Table 1 Dimension and material properties of the elliptical cylindrical shell and piezoelectric stack.

Shell		PZT stack	
Semi-major axis a (mm)	40	Diameter ϕ_c(mm)	10
Semi-minor axis b (mm)	25	Length l_c (mm)	12
Length of shell L_0 (mm)	12	Capacitance C_0 (pF)	633
Thickness h (mm)	2.0	Coupling coefficient k_{33}	0.735
Width of insert d (mm)	10	Dielectric constant ϵ_{33}	10923
Length of insert l_I (mm)	3.0	Compliance s_{33}^D (pm²/N)	9.8
Density ρ (kg/m³)	2710	Density ρ_c (kg/m³)	7700
Young's modulus Y_E(MPa)	68.9	Velocity v (m/s)	3633

3. Results

To analyze the changes in resonant characteristics of the Class IV Flextensional Transducer due to variations in the thickness and aspect ratio of the elliptical cylindrical shell, calculations were based on the transducer specified in **Table 1**. Firstly, the input admittance of the transducer was calculated while varying the thickness h from 1.0 mm to 3.0 mm, as shown in **Fig. 3**. The results indicate that as the thickness of the elliptical cylindrical shell increases, the resonant frequency rises from approximately 3 kHz to around 9 kHz. Furthermore, the electromechanical coupling coefficient decreases for lower frequency resonant modes. To investigate the resonant characteristics according to the aspect ratio of the elliptical cylindrical shell, the thickness h was fixed at 3.0 mm, and the semi-minor axis was varied from 20 mm to 40 mm. The input admittance of the transducer was calculated, as shown in **Fig. 4**. The results indicate that the resonant frequency increases as the aspect ratio a/b increases. In other words, as the shape of the shell, with an elliptical cross-section, approaches a circle, the resonant frequency of the transducer decreases.

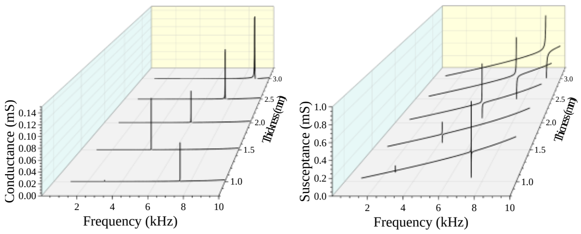
4. Summary

The study analyzed the resonant characteristics of a Class IV Flextensional Transducer by varying the thickness and aspect ratio of its elliptical cylindrical shell. Key findings include:

1) Thickness Variation: Increasing the shell thickness from 1.0 mm to 3.0 mm resulted in an increase in resonant frequency from approximately 3 kHz to around 9 kHz. Additionally, the electromechanical coupling coefficient decreased for lower frequency resonant modes.

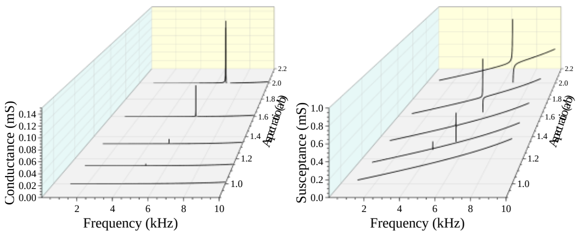
2) Aspect Ratio Variation: With the shell thickness fixed at 3.0 mm, varying the semi-minor axis from 20 mm to 40 mm showed that the resonant frequency increased as the aspect ratio (ratio of semi-major axis to semi-minor axis) increased. This implies that as the shape of the elliptical cross-section of the shell approaches a circle, the resonant frequency decreases.

These findings provide insights into the design elements of Class IV Flextensional Transducers and serve as foundational data for reviewing new designs.



(a) Conductance (b) Susceptance

Fig. 3 Resonant frequency depending on the thickness of the shell.



(a) Conductance (b) Susceptance

Fig. 4 Resonant frequency depending on the aspect ratio of the shell.

Acknowledgment

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References

1) Y. Lam, *Applied Acoustics* **36**, p. 123 (1992).
2) G. Brigham, J. Acoust. Soc. Am., **56**, 31(1974).