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Investigation of Parameter Determination Method for Basis Pursuit Denoising in Underwater Acoustic Communication Using Orthogonal Signal Division Multiplexing

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1. Introduction

Underwater acoustic (UWA) communication offers reliable physical links underwater, where radio and optical waves cannot reach. In addition, UWA communication attracts growing attention as a wireless technology that contributes to the construction of underwater sensor networks. However, realizing reliable UWA communication remains a challenging issue due to the presence of large delay and Doppler spread. To address these challenges, we have proposed a communication scheme, orthogonal signal division multiplexing (OSDM)[1-3], which features a periodic symbol structure in both the time and frequency domain.

Recently, many studies have revealed that utilizing a channel sparsity, another physical characteristic of the UWA channel, improves UWA communication quality[1,3]. For OSDM, an application of basis pursuit denoising (BPDN) to the channel estimation process enables the acquisition of a sparse channel impulse response and enhances communication quality. BPDN recovers a sparse signal by solving the least square problem regularized by ℓ_1 norm and the positive parameter.

Our previous study proposed the global optimum solution (GOS) of BPDN in OSDM and improved demodulation and computational performance[1]. Because GOS derives the solution from the received signal and regularization parameter of BPDN, communication quality heavily depends on determination of the regularization parameter. However, the existing parameter determination process requires a UWA communication simulation to obtain the linear relationship between the noise variance and optimal parameter value, prior to the actual communication[3]. Therefore, introducing a parameter determination method that does not rely on linear relationships would allow BPDN to be performed without prior simulation.

Therefore, this paper proposes a parameter determination method for OSDM using BPDN based on Stein's unbiased risk estimator (SURE)[4]. Moreover, we conducted a sea trial to evaluate the parameter determination performance.

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2. Proposed parameter determination

Let us present the parameter determination procedure for the BPDN in OSDM. The detail signal processing except for parameter determination is described in the reference[1].

The receiver (Rx) obtains a sequence of length $M \in \mathbb{N}$, z_q $(q = -Q, -Q+1, \ldots, Q,\ Q \in \mathbb{N})$ by applying de-spreading matrix to the received signal and estimates the channel impulse response of length M, $\boldsymbol{h}_q$ by solving the following equation,

$$\boldsymbol{z}_q = \boldsymbol{h}_q \boldsymbol{P}_q + \boldsymbol{\eta}_q = \boldsymbol{h}_q \boldsymbol{P} \boldsymbol{R}_q + \boldsymbol{\eta}_q, \qquad (1)$$

where $\boldsymbol{P}$ represents cyclic matrix of the pilot of size $M \times M$, $\boldsymbol{\eta}_q$ is noise vector of length M, $W_{MN} = \exp(1/MN)$, $N \in \mathbb{N}$ and

$$\boldsymbol{R}_q = \mathrm{diag}(W_{MN}^0, W_{MN}^1, \ldots, W_{MN}^{M-1}). \qquad (2)$$

The conventional OSDM (without BPDN) solves the following optimization problem as,

$$\boldsymbol{h}_q^{\mathrm{LS}} = \min_{\boldsymbol{h}_q} \left\| \boldsymbol{z} - \boldsymbol{h}_q \boldsymbol{P}_q \right\|_2^2, \qquad (3)$$

where $\|\cdot\|_u$ represents ℓ_u norm, defined as, $\|\boldsymbol{a}\|_u = \left(\sum_i |a[i]|^u\right)^{1/u}$. On the other hand, the OSDM using BPDN solves the following optimization problem as,

$$\min_{\boldsymbol{h}_q} \left(\left\| \boldsymbol{z} - \boldsymbol{h}_q \boldsymbol{P}_q \right\|_2^2 + \tau_q \left\| \boldsymbol{h}_q \right\|_1 \right), \qquad (4)$$

where $\tau_q \in \mathbb{R}^+$ ($\mathbb{R}^+$ represents a set of positive real number) is regularization parameter. GOS of Eq. (4) is expressed using soft thresholding function as[1],

$$\begin{aligned} h_q^{\mathrm{GOS}}[m] &= \mathrm{Soft}\left(h_q^{\mathrm{LS}}[m], \tau_q\right) \\ &= \max\left(|h_q^{\mathrm{LS}}[m]| - \tau_q, 0\right) \frac{h_q^{\mathrm{LS}}[m]}{|h_q^{\mathrm{LS}}[m]|}. \end{aligned} \qquad (5)$$

Existing system determines the regularization parameter from the linear relationship between the noise deviation σ_{LS} and the optimal parameter value. The Rx obtains this relationship based on the UWA communication simulation in advance. We propose the use of SURE, where the Rx determines the value of τ_q so that the unbiased estimation of error between the unknown $\boldsymbol{h}_q$ and the estimates $\boldsymbol{h}_q^{\mathrm{GOS}}$ is minimized.

Here, we define the isomorphism $\langle \cdot \rangle : \mathbb{C}^M \to \mathbb{R}^{2M}$ as, $\langle \boldsymbol{c} \rangle = (\boldsymbol{a}, \boldsymbol{b}) \in \mathbb{R}^{2M}$ where $\boldsymbol{c} = \boldsymbol{a} + \sqrt{-1}\boldsymbol{b} \in \mathbb{C}^M$. In addition, we introduce $\boldsymbol{g}(\boldsymbol{w}) = \langle \boldsymbol{h}_q^{\mathrm{GOS}} \rangle$, $\boldsymbol{w} = \langle \boldsymbol{h}_q^{\mathrm{LS}} \rangle$, and $\boldsymbol{h} = \langle \boldsymbol{h}_q \rangle$. The Rx determines the regularization parameter value by minimizing mean square error (MSE) between $\boldsymbol{g}(\boldsymbol{w})$ and $\boldsymbol{h}$, as, $\mathbb{E}[||\boldsymbol{g}(\boldsymbol{w})-\boldsymbol{h}||_2^2]$, instead of MSE between $\boldsymbol{h}_q^{\mathrm{GOS}}$ and $\boldsymbol{h}_q$. Under the assumption that $\boldsymbol{\eta}_q$ follows a Gaussian distribution, $\mathbb{E}[||\boldsymbol{g}(\boldsymbol{w})-\boldsymbol{h}||_2^2]$ is calculated as,

$$\mathbb{E}\left[||\boldsymbol{g}(\boldsymbol{w})-\boldsymbol{h}||_2^2\right]=\mathbb{E}[\phi]+||\boldsymbol{h}||_2^2, \qquad (6)$$

$$\varphi=||\boldsymbol{g}(\boldsymbol{w})||_2^2-2\boldsymbol{g}(\boldsymbol{w})\boldsymbol{w}^{\mathrm{T}}+\sigma_{\mathrm{LS}}^2\sum_{i=0}^{2M-1}\frac{dg_i(\boldsymbol{w})}{dw_i}. \qquad (7)$$

Our proposed system determines the regularization parameter value τ_q by minimizing φ.

3. Simulation and Experiment

We conducted a UWA communication simulation using OSDM and ray theory to confirm that φ is an estimate of MSE between $\boldsymbol{h}_q^{\mathrm{GOS}}$ and $\boldsymbol{h}_q$. The simulation environment was set to be the same as the experimental environment. Note that the Tx was moored at a speed of 0 to obtain genuine channel impulse response.

Figure 1 shows the relationship between the regularization parameter and MSE. As shown in the figure, both φ and MSE of $\boldsymbol{h}_q^{\mathrm{GOS}}$ and $\boldsymbol{h}_q$ downwardly convex, indicating that φ worked as an estimate of MSE of $\boldsymbol{h}_q^{\mathrm{GOS}}$ and $\boldsymbol{h}_q$.

We also conducted a sea trial. The experiment was performed in Suruga Bay, Japan. **Fig. 2** shows the experimental environment. The Tx emitted three OSDM signals every one second using a set of signal parameters (M,Q,N) = (127,2,11) and moved at a speed of 2 (m/s). The carrier frequency was 32 kHz, and the effective data rate was 3.2 kbps. The Rx determined the regularization parameter value using the existing linear approximation method (LAM) and the proposed SURE and demodulated messages.

Figures 3 and **4** show the relationship between communication block index and the determined parameter value and the relationship between the Tx-Rx distance and the achieved bit-error rates (BERs), respectively. As shown in Fig. 3, SURE determined a parameter value close to that determined by LAM across all signal blocks. Additionally, as shown in Fig. 4, BPDN using SURE and BPDN using LAM achieved nearly the same BER across all Tx-Rx distance divisions.

4. Conclusion

This paper proposes a new parameter determination method based on SURE for OSDM using BPDN. The obtained results revealed that the proposed SURE determined the regularization parameter close to that obtained by the existing LAM and achieved nearly the same BER of LAM. It is expected that the proposed system is a viable parameter determinator.

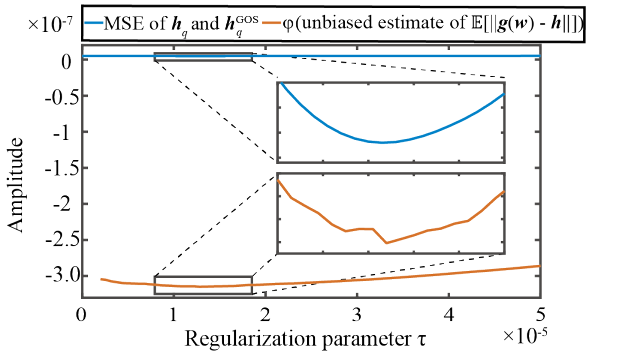

Fig. 1 Relationship between regularization parameter and MSE of $\boldsymbol{h}_q^{\mathrm{GOS}}$ and $\boldsymbol{h}_q$ and φ.

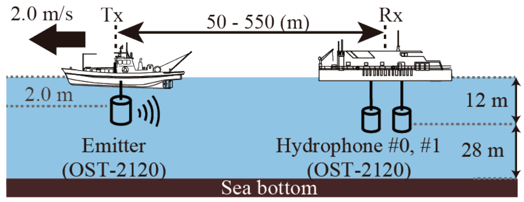

Fig. 2 Experimental environment.

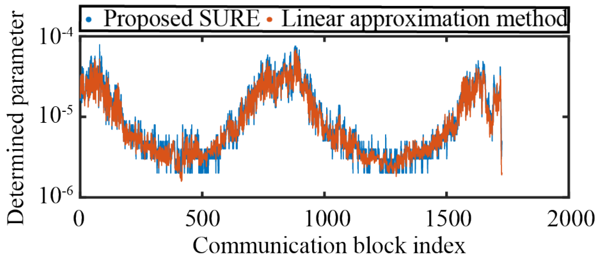

Fig. 3 Relationship between communication block index and determined parameter value.

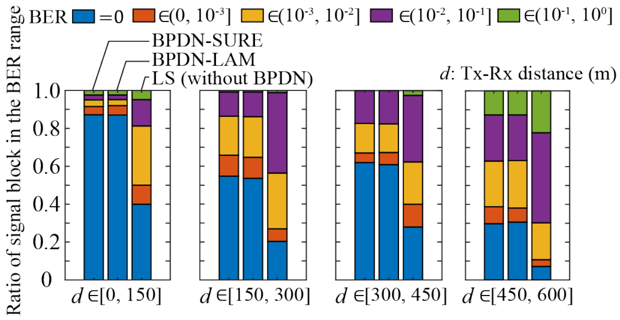

Fig. 4 Relationship between the Tx-Rx distance and achieved BERs.

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