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Analysis of the acoustic field distribution depending on the radius of the circular plate attached to the Langevin transducer

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1. Introduction

In the industrial application of ultrasound, Langevin-type transducers are utilized across a wide range of fields[1)]. Generally, Langevin transducers are used by attaching the vibrating surface to a water tank surface. In this case, the vibrational energy of the ultrasound is transmitted not only to the bottom surface of the tank but also to the sides, resulting in inefficient energy transfer to the acoustic medium. For efficient acoustic energy transfer to the acoustic medium and optimal design for various applications, it is necessary to have an effective method to predict the radiation characteristics delivered to the acoustic medium in the tank. The finite element method, which is widely used for analyzing the acoustic characteristics of transducers, has constraints such as extensive computation time and the requirement for accurate material properties of structures, necessitating a more practical analysis method[2)].

In this study, to improve the radiation characteristics in the acoustic medium of a Langevin-type transducer, we propose a model where a circular plate of a certain area is attached to the radiating surface of the Langevin-type ultrasonic transducer. We derive a solution for the forced vibration, considering the boundary conditions and the radiating surface of the driving force, for a Langevin-type transducer attached to the center of a circular plate with fixed edges.

2. Theoretical analysis model

Figure 1 shows the forced vibration model for a case where a circular plate is attached to the radiating surface of a Langevin-type transducer. A Langevin-type transducer with a radiating surface of radius a_{T} is fixed at the center of a thin circular plate with a fixed edge and radius a, and the driving force $Fe^{j\omega t}$ is applied. The displacement ξ of the forced vibration for the circular plate with fixed edges satisfies the following equation of motion[3)].

$$(\nabla^4 - k^4)\xi = \frac{3(1-\sigma^2)}{2Eh^3} Fe^{j\omega t}. \qquad (1)$$

Here

$$k^4 = \frac{3\rho(1-\sigma^2)}{Eh^2}\omega^2, \qquad (2)$$

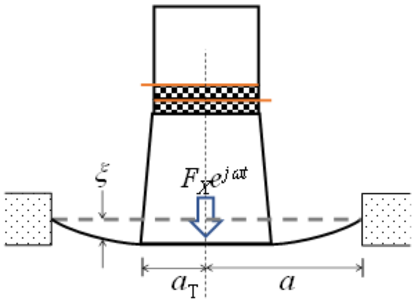


Fig. 1 Vibration model of Langevin ultrasonic transducer attached with fixed rim of the circular plate.

ρ, σ, E and h are the density, Poisson ratio, Young's modulus and the thickness of the circular plate, respectively. F is the amplitude, and ω is the angular frequency of the external force. The boundary conditions in this case are as follows.

$$\xi = 0,\ \frac{d\xi}{dr} = 0,\ at\ r = a. \qquad (3)$$

Before obtaining the solution of Eq. (1), the solution of the free vibration is obtained as follows.

$$\xi = \sum_m \Phi_m \Xi_m \varepsilon^{j\omega_m t}. \qquad (4)$$

Here $y = \frac{r}{a}$ and

$$\Xi_{\mathrm{m}} = J_0(\alpha_m y) - \frac{J_0(\alpha_m)}{I_0(\alpha_m)} I_0(\alpha_m y). \qquad (5)$$

The eigenvalues α_{m} satisfy the following equation and are calculated as shown in **Table 1**.

$$\frac{J_1(ka)}{J_0(ka)} = -\frac{I_1(ka)}{I_0(ka)}. \qquad (6)$$

Table. 1 Eigenvalues of Eq. (1)

m	1	2	3	4	…
$ka = \alpha_{\mathrm{m}}$	3.196	6.306	9.439	12.577	…

By applying the eigenvalues to k in Eq. (2), the natural angular frequencies are obtained as follows.

$$\omega_m = \frac{\alpha_m^2}{a^2}\frac{h\sqrt{E}}{\sqrt{3\rho(1-\sigma^2)}}. \qquad (7)$$

Therefore, the solution for the case when an external force $Fe^{j\omega t}$ is applied to the circular vibrating plate can be expressed as follows

$$\xi = \frac{1}{2h\rho}\sum_m \frac{A_m \Xi_m \varepsilon^{j\omega t}}{\omega_m^2 - \omega^2}. \qquad (8)$$

Here, the coefficient A_m, representing the force distribution of the driving force transferred to the circular plate, can be expressed as follows.

$$A_m = 2\int_x^{x+dx} F\Xi_m y\, dy. \quad (9)$$

Here, F represents a force per unit area.

Regarding the Langevin transducer considered in this study, as shown in Fig. 1, the region where the driving force acts is assumed to be uniformly distributed only within the circle of radius a_T. Therefore, the integration range of Eq. (9) is from a_T to a, and by normalizing with the radius a of the plate, the integration is performed over the range from a_T/a to 1. As a result, the expansion coefficient A_m is obtained as follows.

$$A_m = \frac{2F}{a\alpha_m}\left[B - \frac{I_0(\alpha_m)}{J_0(\alpha_m)}C\right], \quad (10)$$

Here $B = a_1 I_1\left(\frac{\alpha_m a_T}{a}\right) - aI_1(\alpha_m)$, and $C = a_T J_1\left(\frac{\alpha_m a_T}{a}\right) - aJ_1(\alpha_m)$.

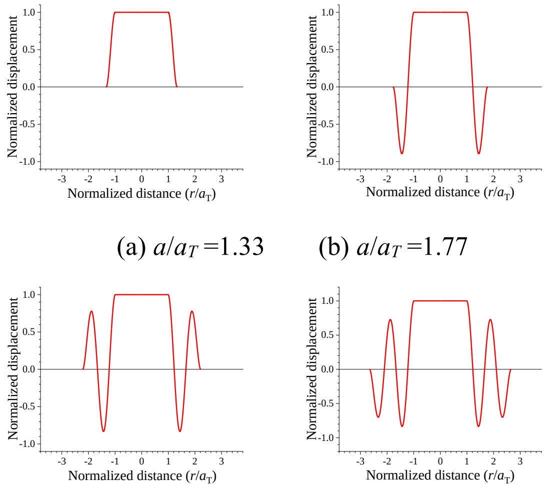
3. Results and discussion

Using the Eq. (8) derived from theoretical analysis, we investigated the vibration displacement distribution and radiation characteristics of a circular plate with fixed edges under forced vibration. The physical properties and thickness settings of the circular vibrating plate in Fig. 1 are summarized in **Table 2**. The radius of the radiating surface of the Langevin transducer was set to a_T=22.5 mm, the driving frequency to f=50 kHz, and the acoustic medium was assumed to be water. The radius of the circular vibrating plate attached to the transducer was set to four different values: a=29.6 mm, 39.5 mm, 49.3 mm, and 59.2 mm, and calculations were performed for each case.

Table. 2 Thickness and physical properties of circular plate.

Thickness (mm)	Density (kg/m^3)	Poisson ratio	Young's modulus (MPa)
1.0	7850	0.3	205

The calculated vibration displacement distribution of the vibrating plate is shown in **Fig. 2**. In the figure, the vertical axis represents the vibration amplitude, and the horizontal axis represents the radial direction of the circular plate. In this study, since we are interested in the shape of the vibration, the magnitude of the vibration displacement has been normalized. We will only consider the vibration modes in the radial direction of the plate. This is because the circular vibrating plate has a relatively large central area fixed as the radiating surface of the Langevin transducer, and the boundary around the plate also has fixed boundary conditions, making it difficult for vibration modes that are not symmetric about the central axis to occur. In Fig. 2, the radial direction is normalized by the radius of the transducer. The area with the red line indicates the region of the circular vibrating plate. Observing the vibration displacement patterns with varying sizes of the vibrating plate, in the range corresponding to the transducer's radius, 0<r/a_T, a uniform displacement is observed due to the constant force applied. However, in the region where r/a_T>1, different bending vibration patterns are exhibited depending on the radius of the circular vibrating plate. Examining Fig. 2(a) for the case where the plate radius a=29.6 mm, the difference between the radius of the transducer a_T and the plate is very small, approximately 7 mm, resulting in a very simple vibration pattern for the plate in its steady state. In Fig. 2(b), where the difference between the plate radius a and the transducer radius a_T is approximately 17 mm, the vibration displacement distribution shows a pattern with half-wavelength characteristics as the vibrating area of the plate increases. In Fig. 2(c) and (d), where the plate area is further increased, the vibration distribution shows patterns with longer wavelengths as the vibrating area increases.



(a) a/a_T=1.33 (b) a/a_T=1.77

(c) a/a_T=2.216 (d) a/a_T=2.66

Fig. 2 Normalized vibration displacement depending on the radius of circular plated.

Acknowledgment

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT) (No. 2022R1F1A1063797).

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