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A study on the relationship between Young's modulus and vibration duration of tuning forks

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1. Introduction

Tuning forks are U-shaped metal instruments that emits a sound of a specific frequency when struck, and are used in variety of fields such as music, medicine, and healing.

The sound produced by a tuning fork is mainly composed of a fundamental tone, emitted from the primary vibration mode, and a harmonic called "clang" tone, with a frequency approximately 6.2 times of the fundamental frequency, produced from the secondary vibration mode (clang mode). Generally, the clang tone decay quickly, while the fundamental tones lasts longer. Considering the purpose of using a tuning fork, the fundamental tone should persist as long as possible and for the clang tone to decay as quickly as possible.

On the other hand, in the manufacturing process of tuning forks, the frequency of the fundamental tone is tuned through processes such as hardening and polishing, however, the duration of the vibration has not been evaluated. Therefore, there are significant individual differences in the sound duration[1)]. Our previous studies have investigated the effects of parameters related to the tuning fork's ambient environment, such as holding strength and temperature on the vibration duration[1,2)]. However, parameters that are expected to influence vibration duration directly, such as the geometry of the tuning fork and material properties like Young's modulus, have not been examined.

Several previous studies have modeled the vibrations of tuning forks. Many of these models represent the tuning fork as a cantilever beam - a rod with one fixed end and the other free - due to the close frequency ratio between the fundamental tone and the overtone produced by the secondary vibration mode [3)]. In this study, assuming that the cantilever beam model can describe the vibrations of the tuning fork, we calculated Young's modulus from the sound generated when tuning forks made of various metallic materials were struck, as well as from the dimensions of the tuning forks. Furthermore, we examined the relationship between Young's modulus and the tuning fork's vibration duration.

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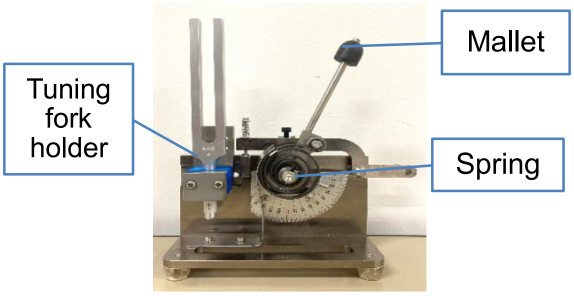

Fig. 1 Tuning fork striking device.

2. Experiments

In this study, we analyzed twenty-six tuning forks: four aluminum tuning forks at 440 Hz, four at 1024 Hz, and two at 528 Hz; four carbon steel tuning forks at 440 Hz and four at 1000 Hz; and four stainless steel tuning forks at 220 Hz, four at 528 Hz.

The tuning forks were struck using an automatic striker (**Fig. 1**) developed in collaboration with Nichion Co., Ltd., a tuning fork manufacturer. The automatic striker consists of a holder for the tuning forks and a mallet drive powered by a mainspring. The force applied by the mallet drive was kept constant throughout the experiment. The striking position was standardized at the node of the "clang" mode, where harmonics are minimized, located between one-quarter and one-third of the leg length from the tip.

The sound produced by the tuning forks was measured using a 1/4-inch free-field microphone (Brüel & Kjær 4939-A-011) and a conditioning amplifier (Brüel & Kjær 2690-0S4). The microphone was positioned at the same height as the striking point of the tuning fork. Measurements were taken in an anechoic chamber, with sufficient distance between the tuning fork and the chamber walls and between the tuning fork and the microphone. The sound before and after striking the tuning forks was recorded for 30 seconds.

Fast Fourier transform (FFT) and short-time Fourier transform (STFT) were applied to the acquired acoustic data to estimate the fundamental frequency. A band-pass filter was then applied to isolate the fundamental tone. The vibration duration was estimated as the time taken for the maximum amplitude value, observed immediately after striking the tuning fork, to decay by a factor of 10. Young's modulus was determined using the following equation (1) based on the eigenfrequency estimated from the sound of the tuning fork and the dimensions of the tuning fork measured with a digital caliper:

$$E = \frac{4\pi^2 f^2 \rho S L^4}{I \alpha^4} \qquad (1)$$

where f is the eigenfrequency, L is the length of the tuning fork, S is the cross-sectional area of the leg, I is the second moment of area, ρ is the density of the tuning fork, and α is a constant determined by the vibration mode (1.875). Young's modulus was calculated using two different length parameters: the leg length of the tuning fork and the total length (leg length + stem length). We then examined the relationship between Young's modulus and the vibration duration.

3. Results

Fig.2 shows the relationship between Young's modulus and the vibration duration of the fundamental tone of each tuning fork. Regardless of the method used to calculate Young's modulus, a consistent trend was observed: higher values of Young's modulus were correlated with shorter vibration durations. This trend was particularly pronounced when Young's modulus was calculated using the total length of the tuning fork. Except for the 220-Hz SUS tuning fork, a slightly strong negative correlation between Young's modulus and vibration duration was identified
($r = -0.657$, $p < 0.001$) (Fig. 2).

When Young's modulus was calculated based on the total length, a significant decrease in vibration duration with increasing Young's modulus was

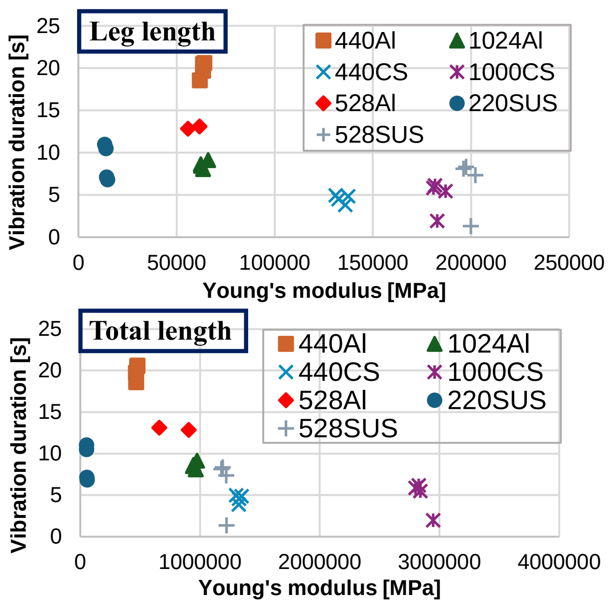

Fig. 2 Young's modulus vs. vibration duration
(Al: Aluminum, SUS: Steel Use Stainless, CS: Carbon steel)

observed for the 1000-Hz carbon steel and the 528-Hz SUS tuning forks ($p < 0.001$).

On the other hand, the 220-Hz SUS tuning fork showed both the lowest Young's modulus and the shortest vibration duration among of all the tuning forks, regardless of the calculation method.

4. Discussion

Except for the 220-Hz SUS tuning fork, vibration duration generally decreased as Young's modulus increased. This finding suggests the possibility of rough estimation of the vibration duration by measuring Young's modulus of tuning forks during manufacturing. More accurate evaluation may be achieved by verifying the accuracy of tuning fork dimensional measurements, the effect of changes in Young's modulus due to thermal expansion of the material, and the accuracy of vibration duration estimation.

The results obtained using both the leg length and the total length of the tuning fork showed the same tendency that vibration duration decreased with increase of Young's modulus, however, the latter effect was more pronounced. In this study, it was assumed that the tuning fork vibration could be described by a cantilever beam model, but the model seems to fit better when only the length of the tuning fork up to the handle is considered. It is also suggested that the handle length has a non-negligible effect on the vibration duration.

On the other hand, the 220-Hz SUS tuning fork showed different results from the other tuning forks, with a smaller Young's modulus but a relatively shorter vibration duration. In general, a smaller Young's modulus tends to increase internal friction, which may result in a more effective dissipation of vibration energy within the tuning fork, resulting to shorter vibration durations. In addition, the extremely short handle of the 220-Hz SUS tuning fork compared to the others may have altered the holding condition of the tuning fork. Further investigation is required to elucidate the cause of this discrepancy.

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