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Elastic-wave circuit with hinge modes in higher-order topological phononic crystals

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1. Introduction

Elastic devices operating at high frequencies have been used for a lot of applications such as a filter of wireless telecommunication devices. Conventional elastic devices have two-dimensional shapes, which lead to the limitations of high integration and complex functions. Three-dimensional (3D) elastic devices are expected to overcome these limitations. While there are many efforts to realize 3D elastic devices, one of the biggest challenges is to make the waveguides perpendicular to the substrate due to the dissipative nature of elastic waves.

Higher-order topological phononic crystals, in which robust boundary modes appear at lower dimensions than bulk, have been attracting attention as systems that are immune against scattering.[1)] Previously we have reported the vertically stacked systems that are topologically nontrivial. These systems have the features of topological hinge states perpendicular to the substrate and edge states parallel to the substrate based on the layer degree of freedom.[2,3)] Moreover, these are quite receptive to fabricate the nanostructures, enabling to make elastic wave devices with high integration and complex functions as shown in **Fig. 1**. However, these topological hinge states are extended from corner states, which gives rise to reflection along the vertical direction.

In this work, we propose a spiral Kagome phononic crystal based on higher-order Weyl semimetals, which inherently have topological hinge states,[4,5)] aiming to develop an efficient waveguide for the vertical propagations.

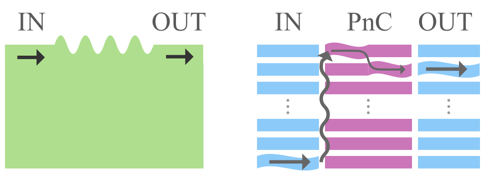

Fig. 1 Schematic illustration of the difference between a conventional elastic device (left) and a vertically stacked 3D elastic device (right).

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2. Spirally stacked Kagome phononic crystal

We construct a tight-binding model of spiral Kagome phononic crystal for obtaining a higher-order Weyl semimetal. Kagome lattice is known for a second-order topological insulator,[6)] when intra-cell hopping t_1 is smaller than inter-cell hopping t_2 as shown in **Fig. 2(a)**. Each layer is stacked spirally, *e.g.*, site A is connected to site B of upper layer and site C of lower layer. This spiral hopping t_3 has an impact on strength of intra-cell hopping. These bonds are not parallel to in-plane direction, consequently intra-cell hopping terms is dependent on the wavenumber of z direction k_z. To modulate the value of t_3, intra-cell hopping is same strength as inter-cell hopping at a certain k_z and eigenstates are degenerated, which is called a Weyl point. Weyl points divide the Brillouin zone into which topological hinge modes appear and disappear.

The band diagram along k_z direction obtained by the tight-binding model is shown in **Fig. 2(b)**. Each hopping term is as follows: $t_1 = -1.0$, $t_2 = -1.3$ and $t_3 = -0.6$. Each band oscillates along k_z and Weyl points appear at wavenumbers marked by red dotted lines.

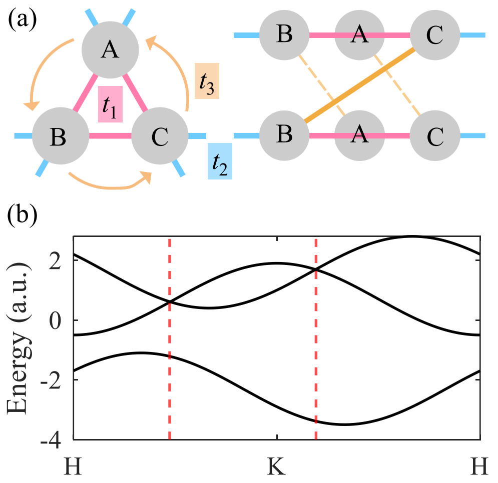

Fig. 2 (a) Schematic illustration of the spiral Kagome phononic crystal. Left panel shows top view and right panel shows side view. (b) The band diagram of the unit cell obtained by tight-binding model. The hopping parameter (t_1, t_2, t_3) = (−1.0, −1.3, −0.6). Red dotted line depicts the wavenumbers where Weyl points emerge.

3. Hinge states protected by topology

Next, we study the supercell with tight binding model to verify the existence of topological hinge states. The band diagram of 10×10 supercell is shown in **Fig. 3(a)**. The hopping parameters are the same as those given in the previous section. The localization at hinges, which is calculated by the existence probability $|\varphi|^2$, is represented by gradations of color. The states concentrating at hinges, fixed at zero energy, emerge near $k_z = \pm(\pi/c)$, while these states are hidden behind bulk states around $k_z = 0$. These hinge states are triply degenerated, corresponding to number of hinges of the triangular supercell.

To validate that these hinge states are derived from the second-order topological feature, we investigate the Wannier center, which determines whether a model is second-order topological insulator in Kagome lattice, along k_z direction. The Wannier center at each k_z is defined as:

$$p_i(k_z) = \frac{1}{S}\iint_{CBZ} A_i \, \mathrm{d}k^2,$$

where CBZ stands for cross-sectional surface at k_z of the Brillouin zone with area S, and A_i is the Berry connection with $i = x$ or y. In our system, (p_x, p_y) takes the discrete values: $\left(0, a/2\sqrt{3}\right)$ or $\left(0, -a/2\sqrt{3}\right)$, the former is nontrivial, and the latter is trivial. The p_x value is constant against any k_z value, therefore we calculate only p_y value. The y coordinate of the Wannier center p_y with various k_z is shown in **Fig. 3(b)**. The value definitely turns the sign at the dotted lines. These lines clearly correspond to the wavenumber where hinge states start to appear. This result manifests that hinge states obtained by the tight-binding model are originated from topological feature of Kagome lattice, being robust against bends, disorder, and defects.

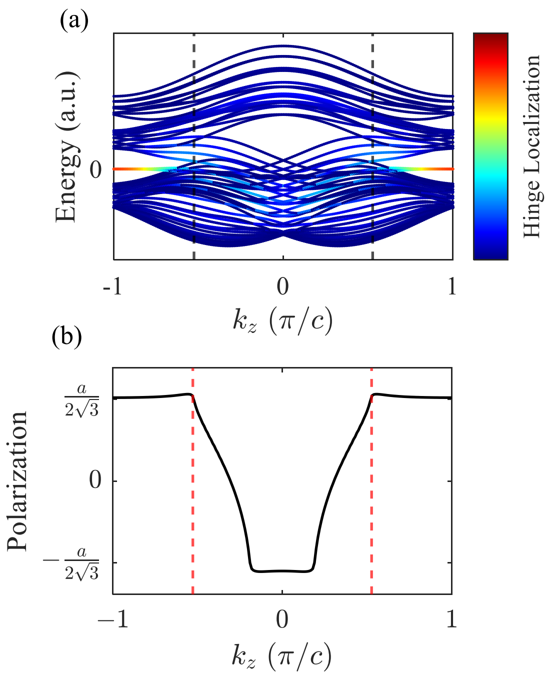

Fig. 3 (a) The band diagram of 10×10 supercell along k_z direction. Color bar denotes the localization at hinges. (b) The Wannier center along k_z direction. The Wannier center takes the value $+a/2\sqrt{3}$ in nontrivial state and $-a/2\sqrt{3}$ in trivial state. In each figure, dotted lines represent the wavenumbers where the Weyl points emerge.

4. Conclusion

We proposed a spiral Kagome phononic crystal based on a higher-order topological Weyl semimetal to acquire topological hinge states propagating without scattering. First, we built a tight-binding model of spiral Kagome phononic crystal, and it is found that Weyl points appeared with appropriate values of hopping terms. Next, we constructed the supercell with tight-binding model. The result showed that hinge states emerge in the limited k_z region. Finally, we calculated the Wannier center for verifying these hinge states is topologically protected. The structure we proposed is dependent on k_z value, enabling to design applications such as waveguides that is selective for the incident angles of elastic waves.

Acknowledgment

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